ECE 3640 - Discrete-Time Signals and Systems
Windowed Filter Design

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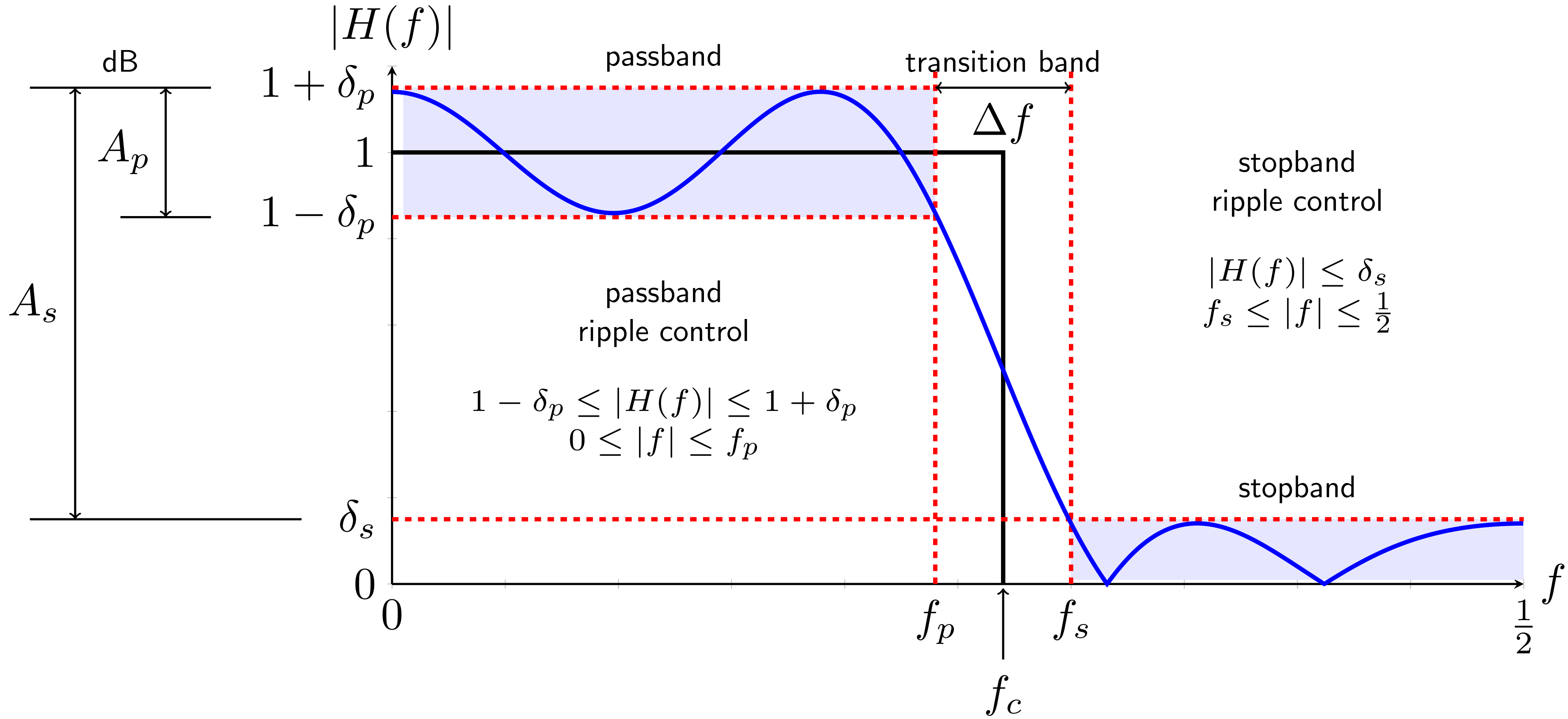
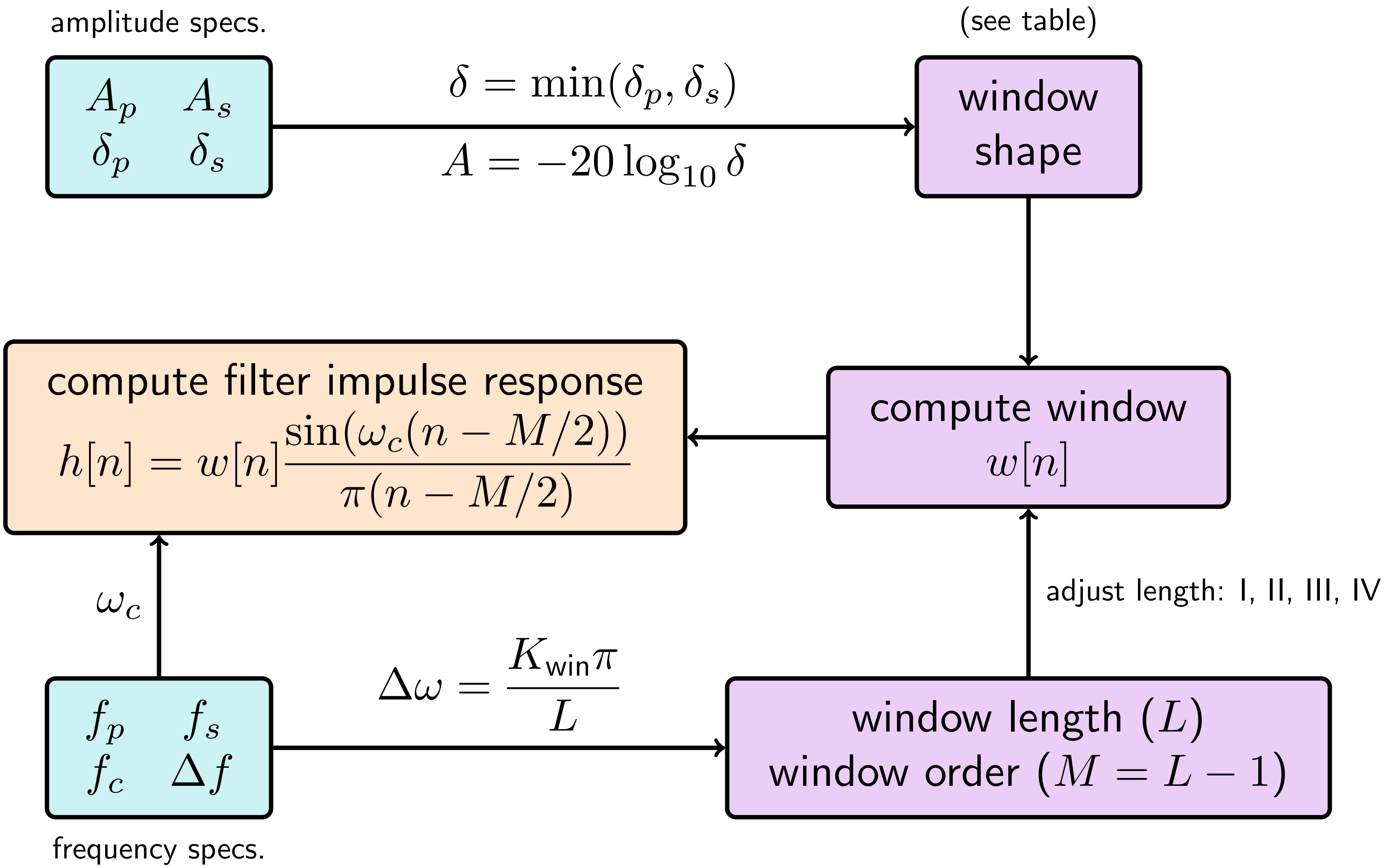
Lowpass Filter Specification



$$\delta_p = \frac{\gamma - 1}{\gamma + 1}, \gamma = 10^{A_p/20} \qquad A_p = 20\log_{10}\left(\frac{1+\delta_p}{1-\delta_p}\right) \qquad f_c = \frac{1}{2}(f_s + f_p)$$

$$\delta_s = 10^{-A_s/20} \qquad A_s = 20\log_{10}\left(\frac{1+\delta_p}{\delta_s}\right) \qquad \Delta f = f_s - f_p$$

Lowpass Filter Design Procedure

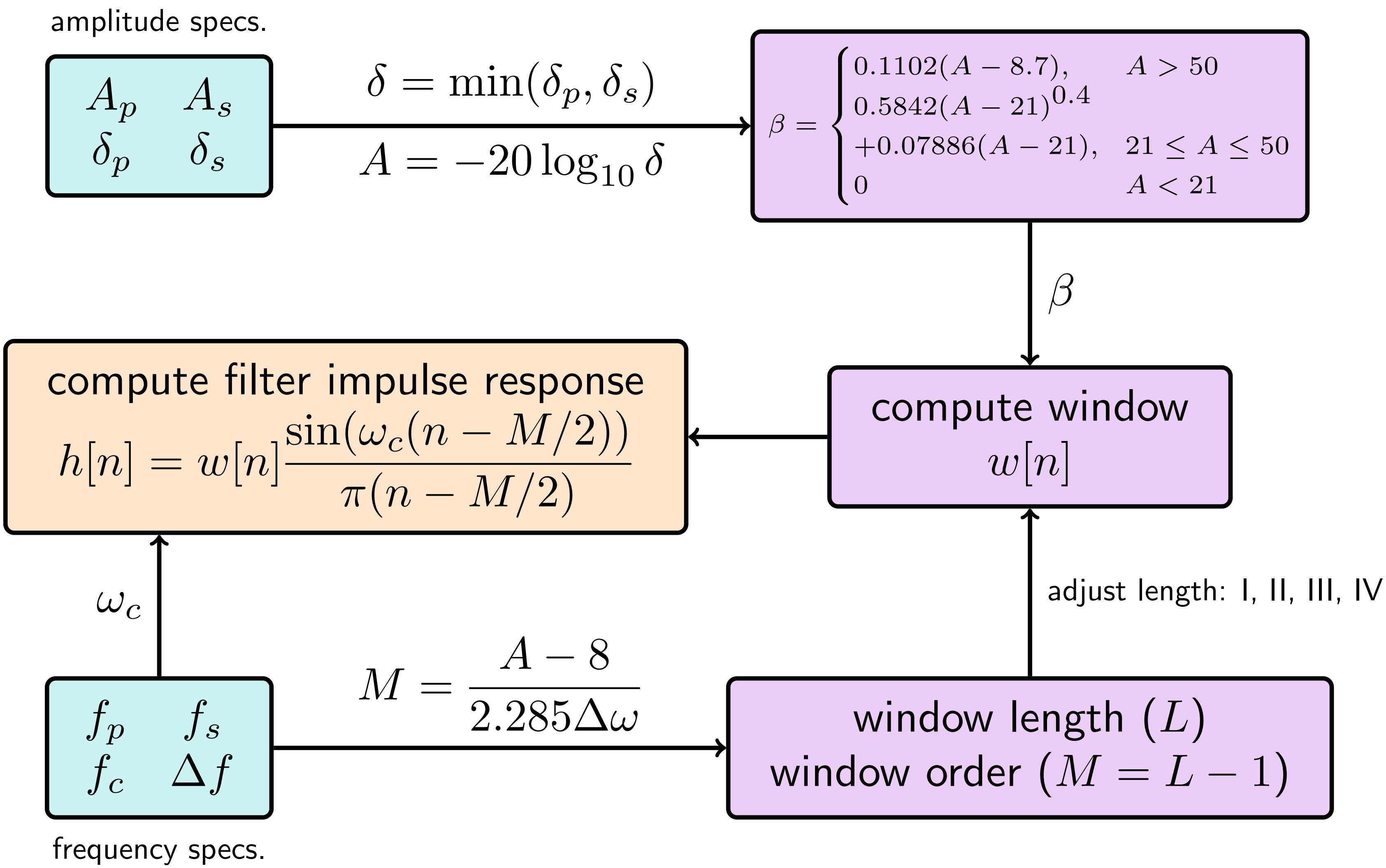


Does the filter meet the specifications? Check $|H(f)|$.
If not, increase length and check again.

Table

window name	side lobe level (dB)	approx. $\Delta\omega$	exact $\Delta\omega$	$\delta_p \approx \delta_s$	A_p (dB)	A_s (dB)
rectangular	-13	$\frac{4\pi}{L}$	$\frac{1.8\pi}{L}$	0.09	0.75	21
Bartlett	-25	$\frac{8\pi}{L}$	$\frac{6.1\pi}{L}$	0.05	0.45	26
Hann	-31	$\frac{8\pi}{L}$	$\frac{6.2\pi}{L}$	0.0063	0.055	44
Hamming	-41	$\frac{8\pi}{L}$	$\frac{6.6\pi}{L}$	0.0022	0.019	53
Blackman	-57	$\frac{12\pi}{L}$	$\frac{11\pi}{L}$	0.0002	0.0017	74

Lowpass Filter Design Procedure: Kaiser Window



Does the filter meet the specifications? Check $|H(f)|$.
If not, increase length and check again.

Kaiser Window

$$w[n] = \frac{I_0\left(\beta\left(1-\left[\frac{n-\alpha}{\alpha}\right]^2\right)^{\frac{1}{2}}\right)}{I_0(\beta)}$$

$$0 \leq n \leq M = L-1, \qquad \alpha = M/2 = \text{group delay}$$

$I_0(x)$ is the zero-th order modified Bessel function of the first kind