ECE 3640 - Discrete-Time Signals and Systems
Systems in the Time Domain

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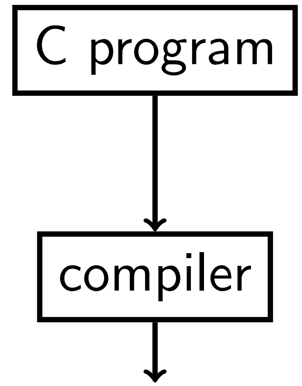
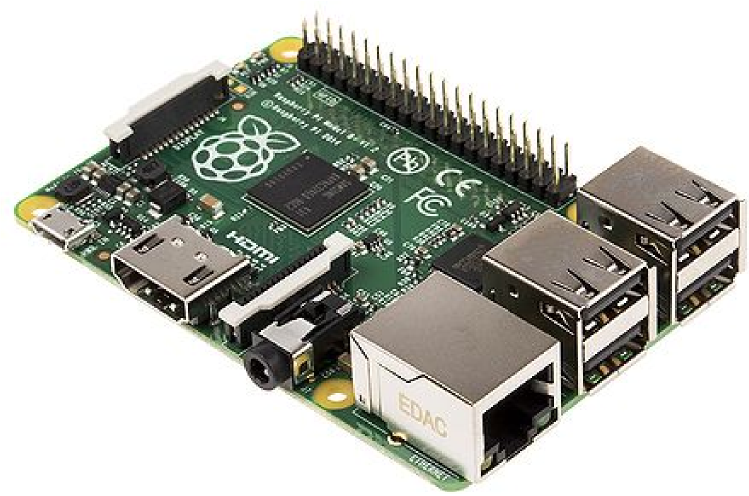
analog and digital systems

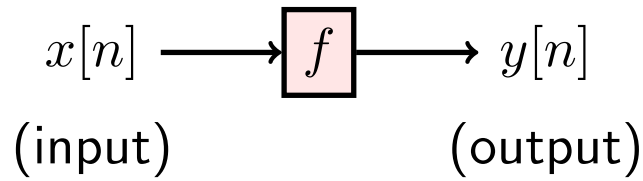
analog circuit
(continuous-time system)

digital circuit
(discrete-time system)





discrete-time systems



mathematical definition:

$$\begin{aligned} y[n] &= f\left(\{x[i]\}_{i=\infty}^{\infty}; n\right) \\ &= f(\underbrace{\cdots, x[n-2], x[n-1], x[n], x[n+1], x[n+2], \cdots}_{\text{the whole input signal}}; \underbrace{n}_{\text{output time}}) \end{aligned}$$

f can be any function that can conceivably be implemented on a computer

example discrete-time systems

$y[n] = x[n]$	identity system
$y[n] = ax[n]$	amplifier
$y[n] = x[n-d]$	d sample delay
$y[n] = \frac{1}{3}\left(x[n] + x[n-1] + x[n-2]\right)$	moving average (MA) 1
$y[n] = \frac{1}{3}\left(x[n+1] + x[n] + x[n-1]\right)$	moving average (MA) 2
$y[n] = \text{median}(x[n+1], x[n], x[n-1])$	median filter
$y[n] = x[n] + x[n-1] + x[n-2] + \cdots$	integrator/accumulator
$\quad = x[n] + y[n-1]$	recursive form of accumulator

$$= \sum_{k=0}^{\infty} x[n-k] = \sum_{k=-\infty}^{n} x[k]$$

example discrete-time systems

$y[n] = |x[n]|$ rectifier

$y[n] = x[Dn]$ compressor

$$y[n] = \begin{cases} x[\frac{n}{U}], & n = kU + i, i = 0 \\ 0, & n = kU + i, i = 1, 2, \cdots, U - 1 \end{cases}$$ expander

$y[n] = x[n]\cos(2\pi f n)$ moduator

causal systems

$$y[n] = f(\underbrace{\cdots, x[n-2], x[n-1], x[n]}_{\text{current and past input samples}}; \quad \underbrace{n}_{\text{output time}} \quad)$$

- response cannot appear before excitation
- non-anticipatory
- causality required for real-time systems
- non-causal processing possible off-line

causal systems

$y[n] = x[n]$	causal
$y[n] = ax[n]$	causal
$y[n] = x[n-d]$	causal if $d \geq 0$
$y[n] = x[n-d]$	non-causal if $d < 0$
$y[n] = \frac{1}{3}(x[n] + x[n-1] + x[n-2])$	causal
$y[n] = \frac{1}{3}(x[n+1] + x[n] + x[n-1])$	non-causal
$y[n] = \text{median}(x[n+1], x[n], x[n-1])$	non-causal
$y[n] = x[n] + x[n-1] + x[n-2] + \cdots$	causal
$y[n] = \|x[n]\|$	causal
$y[n] = x[n]\cos(2\pi f n)$	causal

BIBO stable systems

$x[n]$ is bounded if $|x[n]| \leq M < \infty$ for all n

bounded input/bounded output (BIBO) stable system: bounded inputs lead to bounded outputs

$$y[n] = \frac{1}{3}\left(x[n] + x[n-1] + x[n-2]\right) \qquad \text{stable}$$

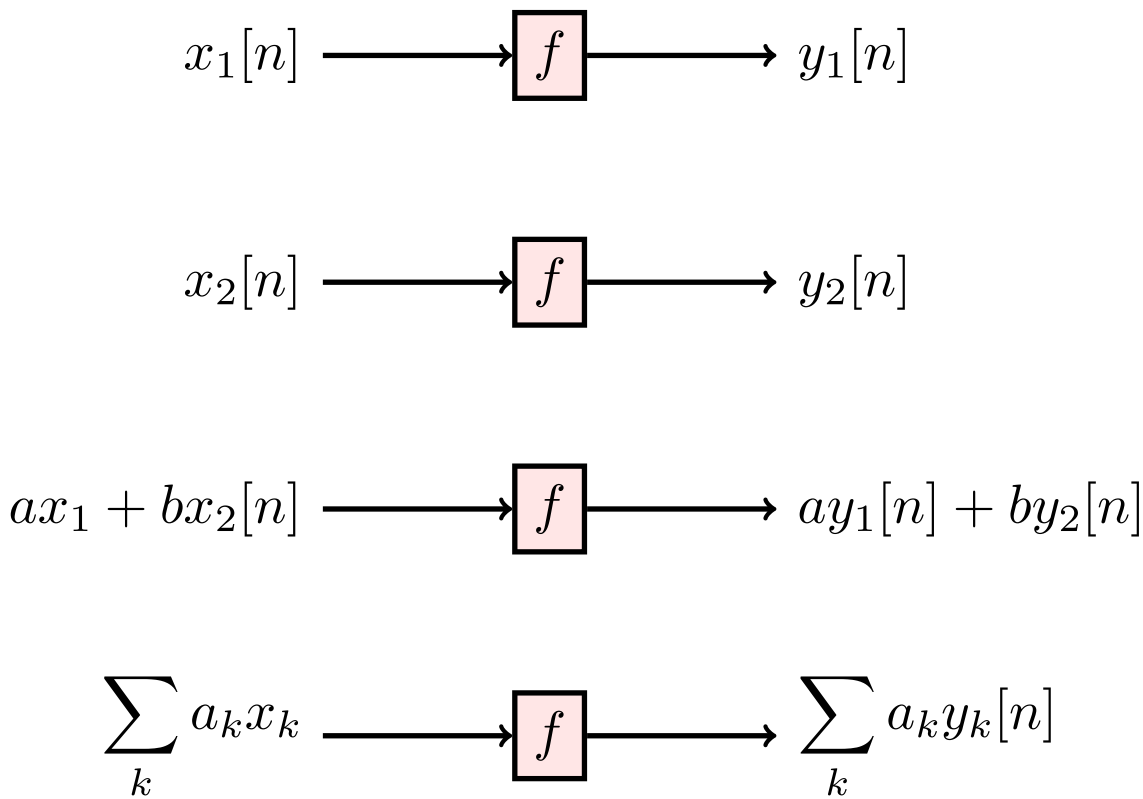
$$y[n] = \frac{1}{3}\left(x[n+1] + x[n] + x[n-1]\right) \qquad \text{stable}$$

$$y[n] = \text{median}(x[n+1], x[n], x[n-1]) \qquad \text{stable}$$

$$y[n] = x[n] + x[n-1] + x[n-2] + \cdots \qquad \text{unstable}$$

Put $x[n] = u[n]$ (bounded) into the accumulator, and $y[n] = (n+1)u[n]$ (unbounded) comes out.

linear systems

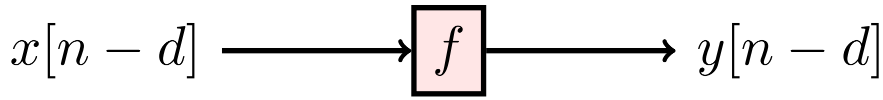


- principle of superposition holds
- zero in leads to zero out

linear systems

$y[n] = x[n]$	linear
$y[n] = ax[n]$	linear
$y[n] = x[n-d]$	linear
$y[n] = \frac{1}{3}\left(x[n] + x[n-1] + x[n-2]\right)$	linear
$y[n] = \frac{1}{3}\left(x[n+1] + x[n] + x[n-1]\right)$	linear
$y[n] = \text{median}(x[n+1], x[n], x[n-1])$	non-linear
$y[n] = x[n] + x[n-1] + x[n-2] + \cdots$	causal
$y[n] = \lvert x[n] \rvert$	non-linear
$y[n] = x[n]\cos(2\pi f n)$	linear

time invariance



(for all n and for all shifts d)

- characteristics of the system do not change with time
- shape of output depends on the shape of the input but not on the time instant at which the input is applied

$y[n] = x[n-d]$	time invariant
$y[n] = \frac{1}{3}\left(x[n+1] + x[n] + x[n-1]\right)$	time invariant
$y[n] = \text{median}(x[n+1], x[n], x[n-1])$	time invariant
$y[n] = x[n] + x[n-1] + x[n-2] + \cdots$	time invariant
$y[n] = \lvert x[n] \rvert$	time invariant
$y[n] = x[n]\cos(2\pi f n)$	time varying

summary

We are mainly interested in causal, stable, linear, time-invariant systems.

Linear, time-invariant systems are called LTI systems.