ECE 3640 - Discrete-Time Signals and Systems
Signals in the Time Domain

Jake Gunther

UtahStateUniversity

Department of Electrical & Computer Engineering

signal definition

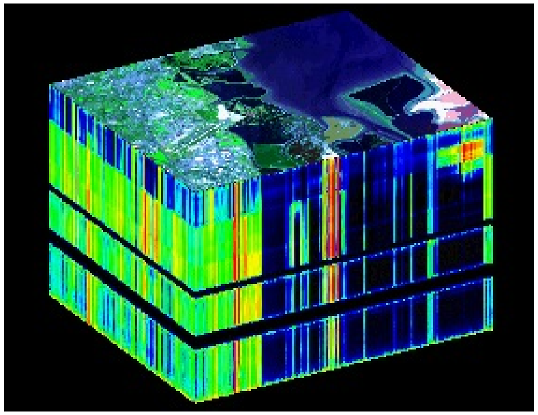
signal definition

Signal = Something that conveys information. Information conveyed in patterns of variation.

Signal = Mathematical function with domain (independent variables) and range (dependent variables).

$f(t)$	monaural sound wave, radio signal
$\begin{bmatrix} f_{\text{left}}(t) \\ f_{\text{right}}(t) \end{bmatrix}$	stereo sound wave
$f(x,y)$	grayscale picture of a scene
$\begin{bmatrix} f_{\text{red}}(x,y) \\ f_{\text{green}}(x,y) \\ f_{\text{blue}}(x,y) \end{bmatrix}$	RGB color picture of a scene
$f(x,y,t)$	silent film (grayscale)

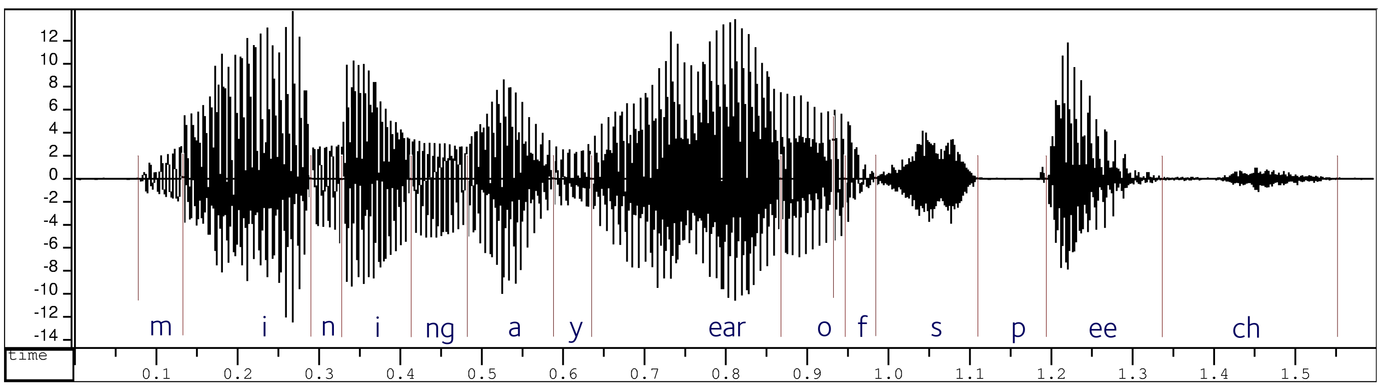
signal examples



hyperspectral image

synthetic aperture radar image

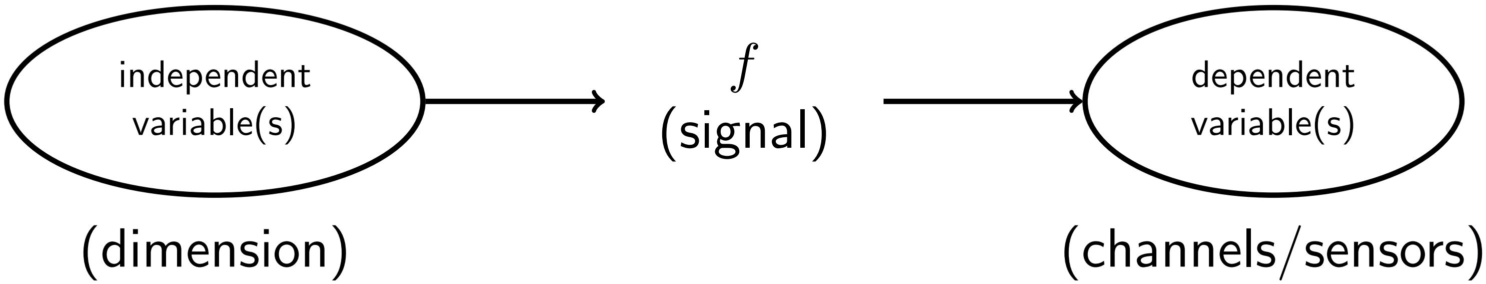
MRI image



speech signal

electrocardiogram

signal attributes



Dimension = number of independent variables of a signal

Channels = number of dependent variables of a signal, number of sensors

$$
\begin{array}{rcll}
f(t) & \rightarrow & \text{1 dim.} & \text{1 chan.} \\
\begin{bmatrix} f_{\text{left}}(t) \\ f_{\text{right}}(t) \end{bmatrix} & \rightarrow & \text{1 dim.} & \text{2 chan.} \\
f(x,y) & \rightarrow & \text{2 dim.} & \text{1 chan.} \\
\begin{bmatrix} f_{\text{red}}(x,y) \\ f_{\text{green}}(x,y) \\ f_{\text{blue}}(x,y) \end{bmatrix} & \rightarrow & \text{2 dim.} & \text{3 chan.} \\
f(x,y,t) & \rightarrow & \text{3 dim.} & \text{1 chan.}
\end{array}
$$

variables

Independent variable(s):

- Continuous, $f(t), t \in \mathbb{R}$ (real numbers, the physical world, time, space, etc.)
- Discrete, $f[n], n \in \mathbb{Z}$ (integers, discrete or sampled data, cyberspace)

Note: Sampling (A/D) and reconstruction (D/A) provides interface between continuous and discrete realms

Dependent variable(s):

- Continuous (analog)
  - physical world
  - ex: voltage in a circuit
- Discrete (digital)
  - physical world, ex: number of eggs laid by a chicken each day
  - sampled data world, ex: 8-bit per pixel grayscale digital image
  - ex: sampled audio signal with 8-bit quantization (256 levels)
  - ex: sampled audio signal with 24-bit quantization (16 million levels)

domain and range

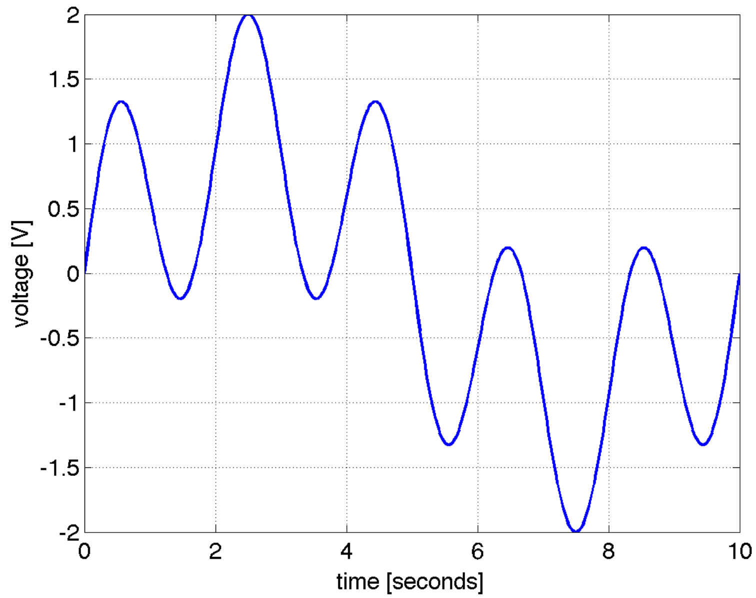
domain:

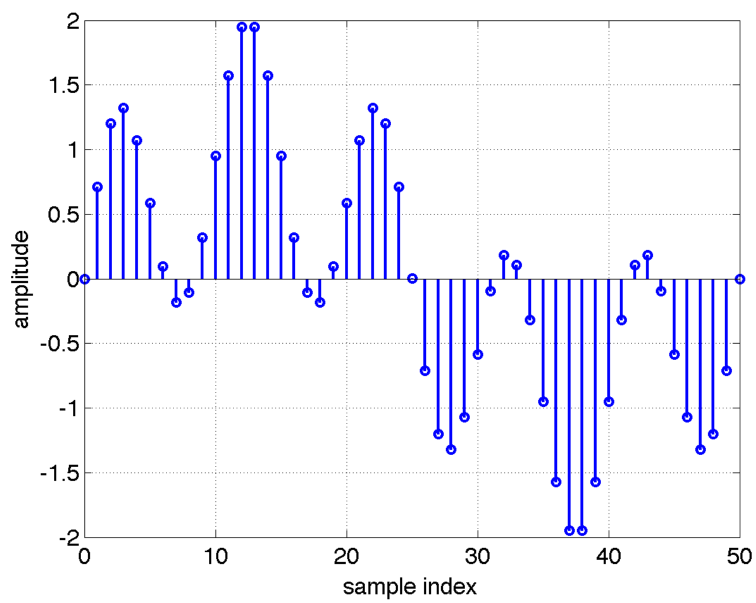
- continuous-time, $t \in \mathbb{R}$ [seconds] (uncountable)
- continuous-space, $x, y, z \in \mathbb{R}$ [meters]
- continuous-wavelength, $\lambda \in \mathbb{R}$ [meters]
- discrete-time, $n \in \mathbb{Z}$ [unitless] (countable)

range:

- $f(t) \in \mathbb{R}$, ex: voltage in a circuit, pressure in air
- $f(t) \in \mathbb{C}$, ex: complex baseband radio signal
- $f(t) \in \mathbb{A}$ (finite alphabet), ex: symbols, words, etc.

range of discrete-time signals





$x[\ 0]$	$=$	0.00000	$x[25]$	$=$	0.00000
$x[\ 1]$	$=$	0.71312	$x[26]$	$=$	-0.71312
$x[\ 2]$	$=$	1.19975	$x[27]$	$=$	-1.19975
$x[\ 3]$	$=$	1.31918	$x[28]$	$=$	-1.31918
$x[\ 4]$	$=$	1.06954	$x[29]$	$=$	-1.06954
$x[\ 5]$	$=$	0.58779	$x[30]$	$=$	-0.58779
$x[\ 6]$	$=$	0.09676	$x[31]$	$=$	-0.09676
$x[\ 7]$	$=$	-0.18054	$x[32]$	$=$	0.18054
$x[\ 8]$	$=$	-0.10673	$x[33]$	$=$	0.10673
$x[\ 9]$	$=$	0.31704	$x[34]$	$=$	-0.31704
$x[10]$	$=$	0.95106	$x[35]$	$=$	-0.95106
$x[11]$	$=$	1.57007	$x[36]$	$=$	-1.57007
$x[12]$	$=$	1.94908	$x[37]$	$=$	-1.94908
$x[13]$	$=$	1.94908	$x[38]$	$=$	-1.94908
$x[14]$	$=$	1.57007	$x[39]$	$=$	-1.57007
$x[15]$	$=$	0.95106	$x[40]$	$=$	-0.95106
$x[16]$	$=$	0.31704	$x[41]$	$=$	-0.31704
$x[17]$	$=$	-0.10673	$x[42]$	$=$	0.10673
$x[18]$	$=$	-0.18054	$x[43]$	$=$	0.18054
$x[19]$	$=$	0.09676	$x[44]$	$=$	-0.09676
$x[20]$	$=$	0.58779	$x[45]$	$=$	-0.58779
$x[21]$	$=$	1.06954	$x[46]$	$=$	-1.06954
$x[22]$	$=$	1.31918	$x[47]$	$=$	-1.31918
$x[23]$	$=$	1.19975	$x[48]$	$=$	-1.19975
$x[24]$	$=$	0.71312	$x[49]$	$=$	-0.71312

- CT signal is a waveform
- DT signal is a sequence of numbers (concept of time encoded in sample rate)

range of digital signals

- DT and digital signals: concept of time is lost

- DT and digital signals are undefined between samples (not zero)

- need sample rate to interpret DT and digital signals

- DT signals have “amplitude”

- digitals signals are binary codes

$$x_d[n] = b_1 2^{-1} + b_2 2^{-2} + \cdots + b_B 2^{-B},$$
$$x[n] \approx R\, x_d[n],$$

where R is a reference voltage

$x_d[\ 0]$	=	0011001000	$x_d[25]$	=	0011001000
$x_d[\ 1]$	=	0100001111	$x_d[26]$	=	0010000001
$x_d[\ 2]$	=	0101000000	$x_d[27]$	=	0001010000
$x_d[\ 3]$	=	0101001100	$x_d[28]$	=	0001000100
$x_d[\ 4]$	=	0100110011	$x_d[29]$	=	0001011101
$x_d[\ 5]$	=	0100000011	$x_d[30]$	=	0010001101
$x_d[\ 6]$	=	0011010010	$x_d[31]$	=	0010111110
$x_d[\ 7]$	=	0010110110	$x_d[32]$	=	0011011010
$x_d[\ 8]$	=	0010111101	$x_d[33]$	=	0011010011
$x_d[\ 9]$	=	0011101000	$x_d[34]$	=	0010101000
$x_d[10]$	=	0100100111	$x_d[35]$	=	0001101001
$x_d[11]$	=	0101100101	$x_d[36]$	=	0000101011
$x_d[12]$	=	0110001011	$x_d[37]$	=	0000000101
$x_d[13]$	=	0110001011	$x_d[38]$	=	0000000101
$x_d[14]$	=	0101100101	$x_d[39]$	=	0000101011
$x_d[15]$	=	0100100111	$x_d[40]$	=	0001101001
$x_d[16]$	=	0011101000	$x_d[41]$	=	0010101000
$x_d[17]$	=	0010111101	$x_d[42]$	=	0011010011
$x_d[18]$	=	0010110110	$x_d[43]$	=	0011011010
$x_d[19]$	=	0011010010	$x_d[44]$	=	0010111110
$x_d[20]$	=	0100000011	$x_d[45]$	=	0010001101
$x_d[21]$	=	0100110011	$x_d[46]$	=	0001011101
$x_d[22]$	=	0101001100	$x_d[47]$	=	0001000100
$x_d[23]$	=	0101000000	$x_d[48]$	=	0001010000
$x_d[24]$	=	0100001111	$x_d[49]$	=	0010000001

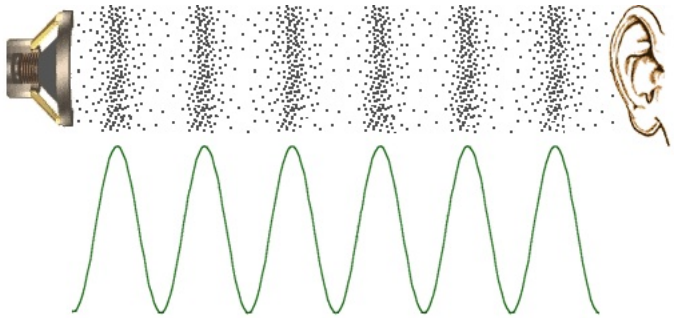
signal definition summary

term	definition
signal	something that conveys information in patterns of variation
dimension	number of independent variables
channels	number of dependent variables
continuous "time"	continuous independent variables
discrete "time"	discrete independent variables
analog	continuous dependent variables
digital	discrete dependent variables

signal examples

signal examples

sound wave



radio wave

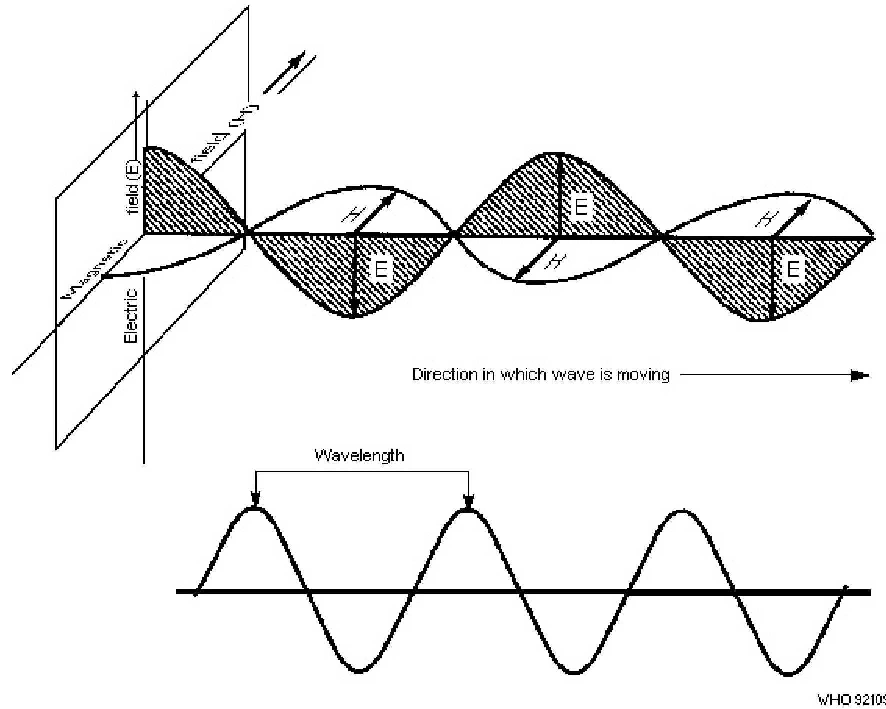
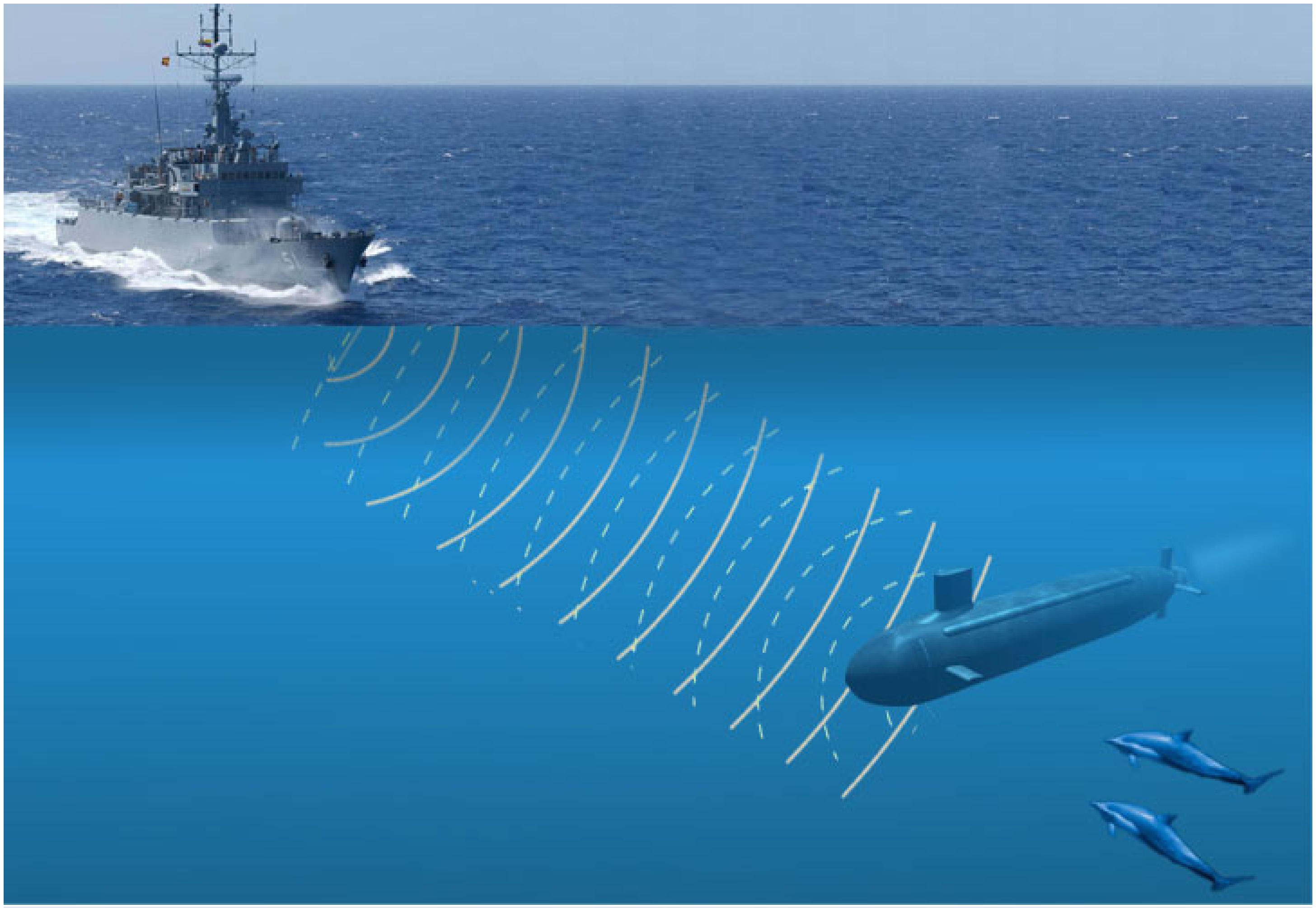


Fig. 1. An electromagnetic monochromatic wave. Electromagnetic waves consist of electrical and magnetic forces that move in consistent wave-like patterns at right angles to each other for far-field propagation, but at varying angles in the near-field.

sound wave
image/scene

signal example: sonar

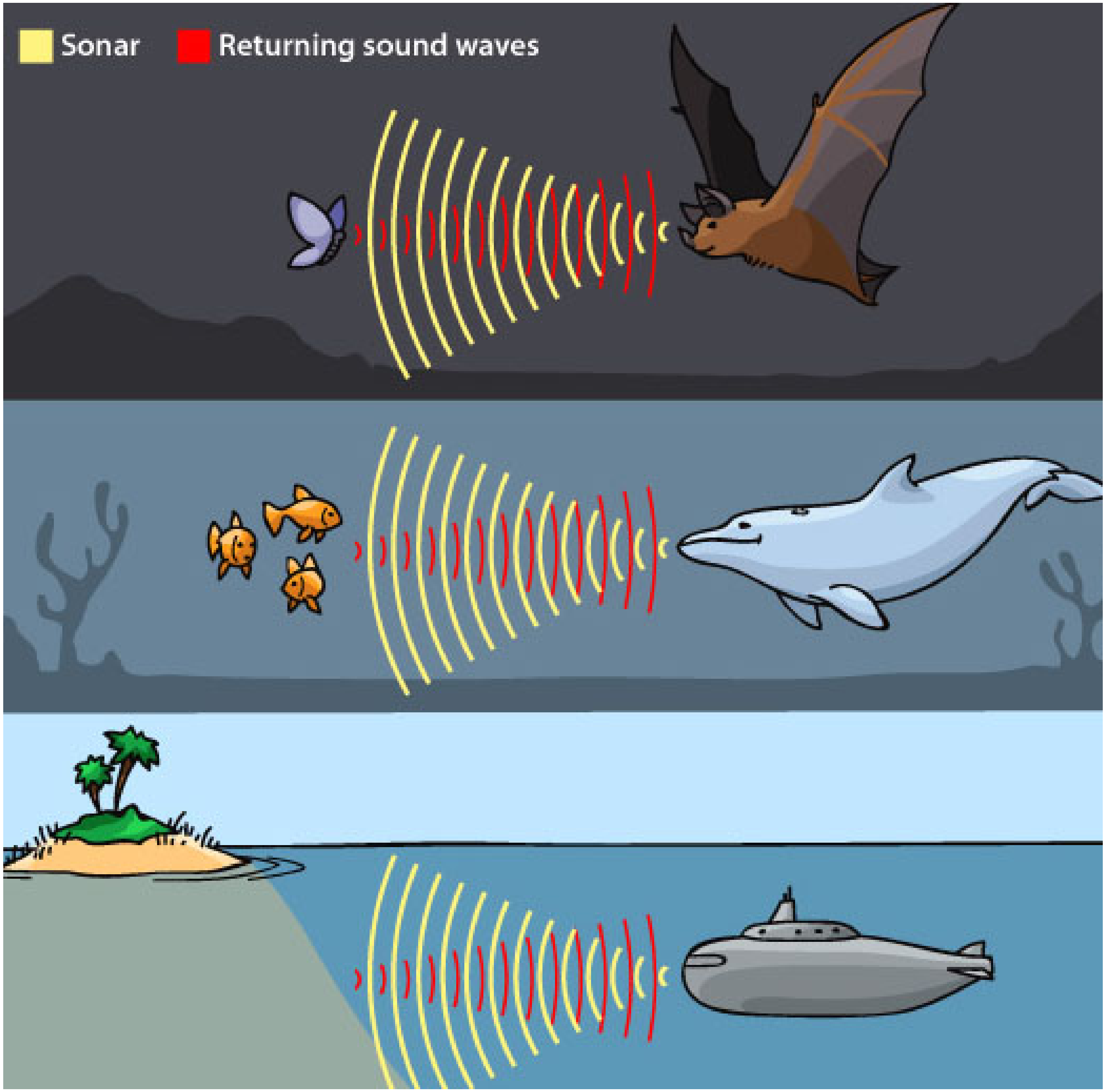
- How do patterns convey information?
- What are the signal attributes?



Warships using sonar to find submarines. It's thought the war ships sonar disrupts dolphins ultrasound system.

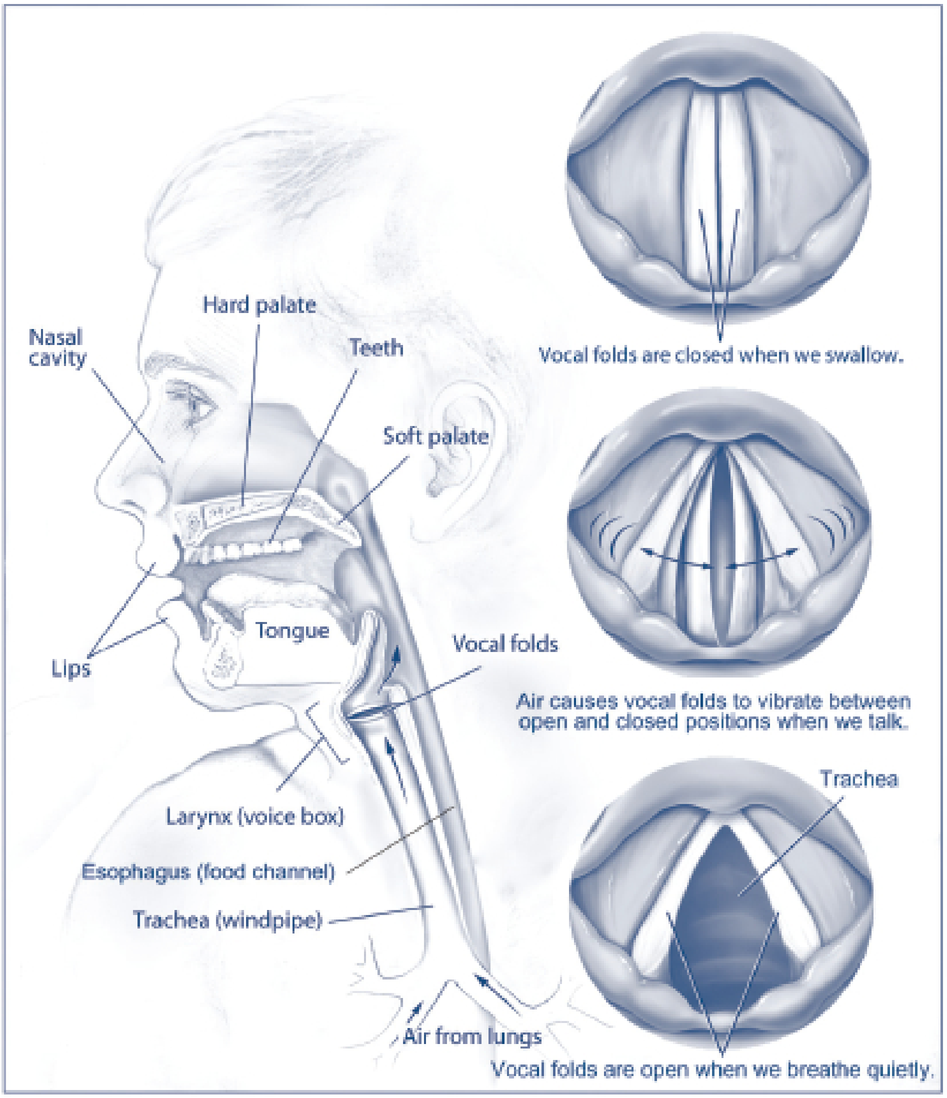
signal example: echolocation

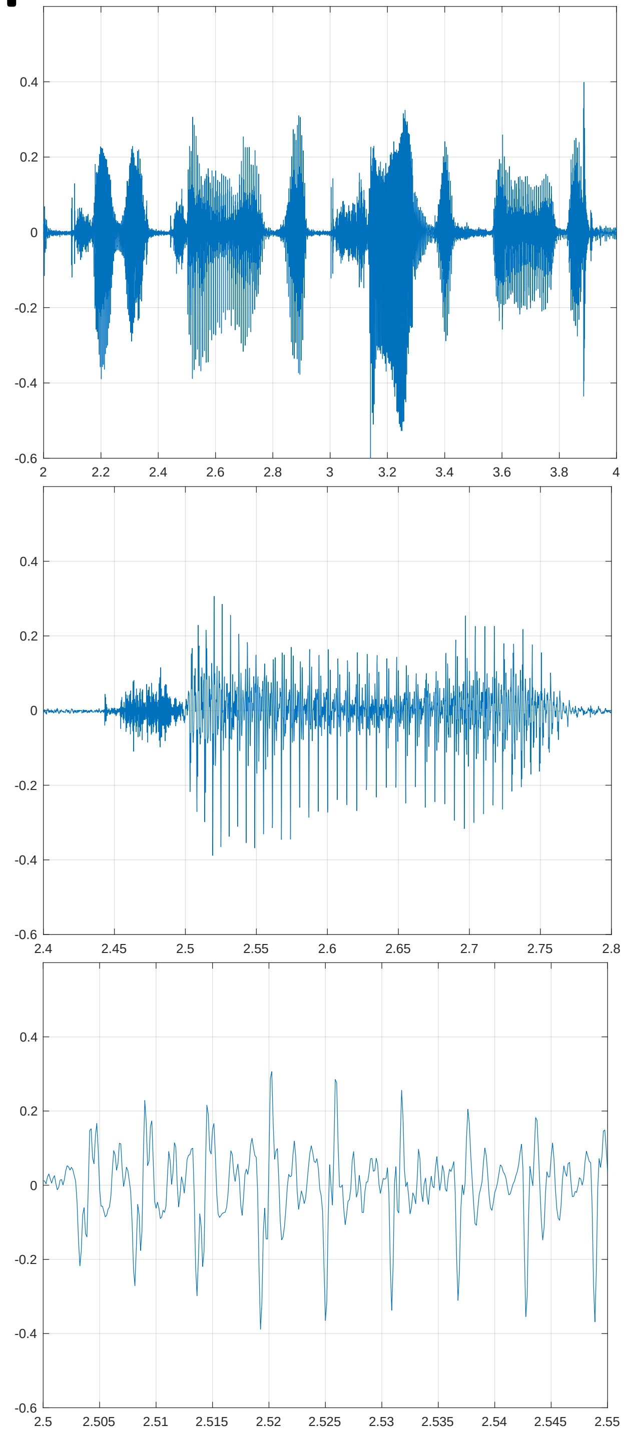
- How do patterns convey information?
- What are the signal attributes?



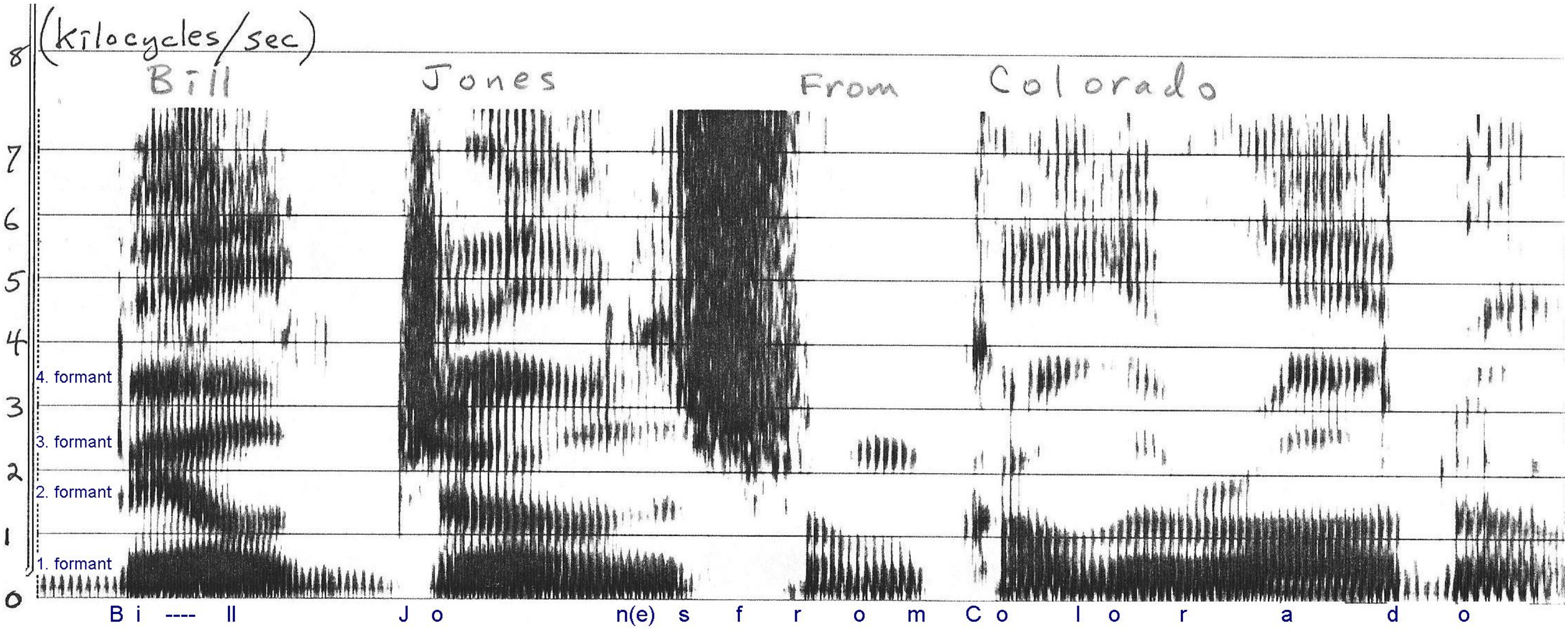
signal example: speech

- How do patterns convey information?
- What are the signal attributes?

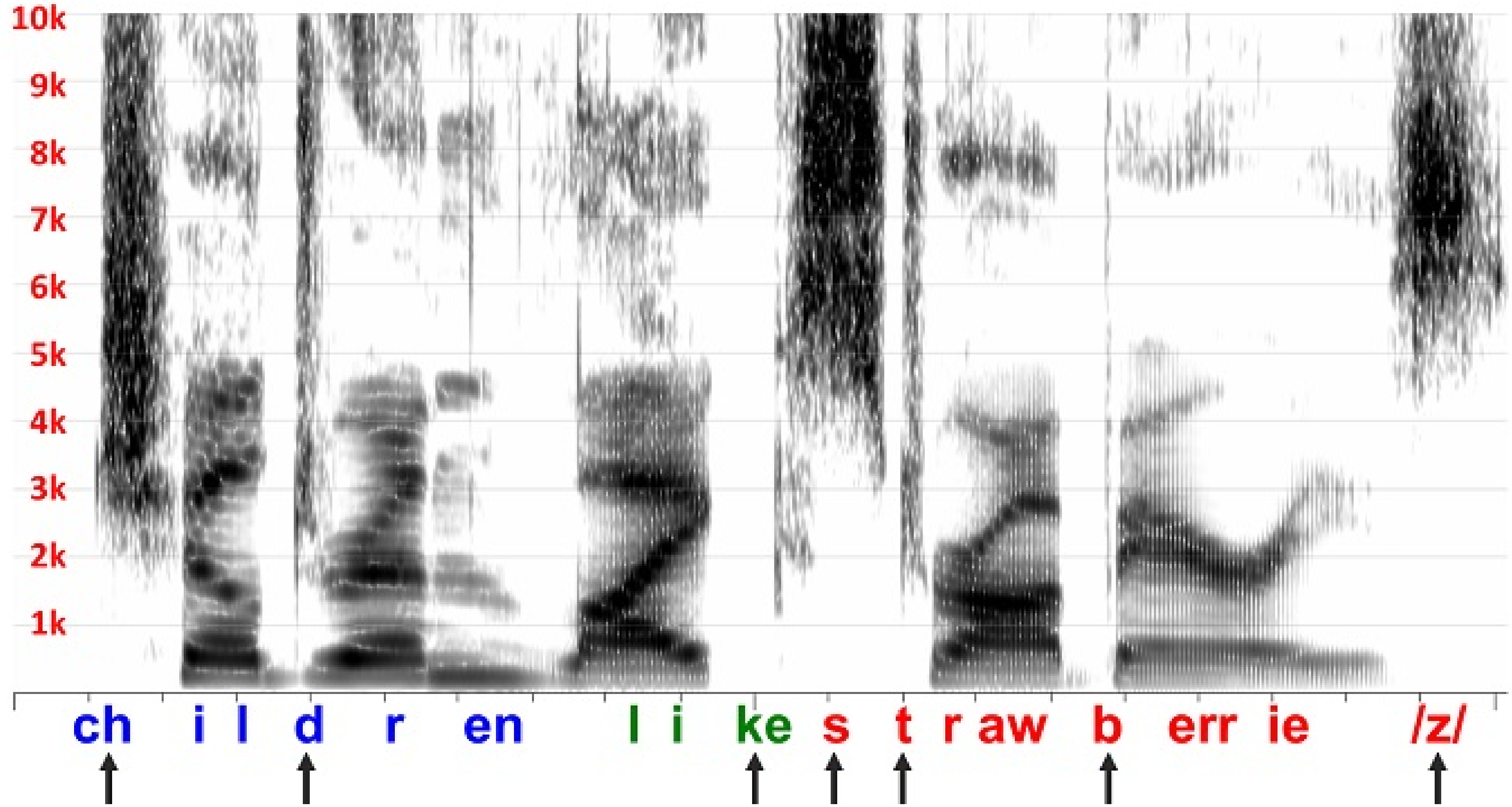




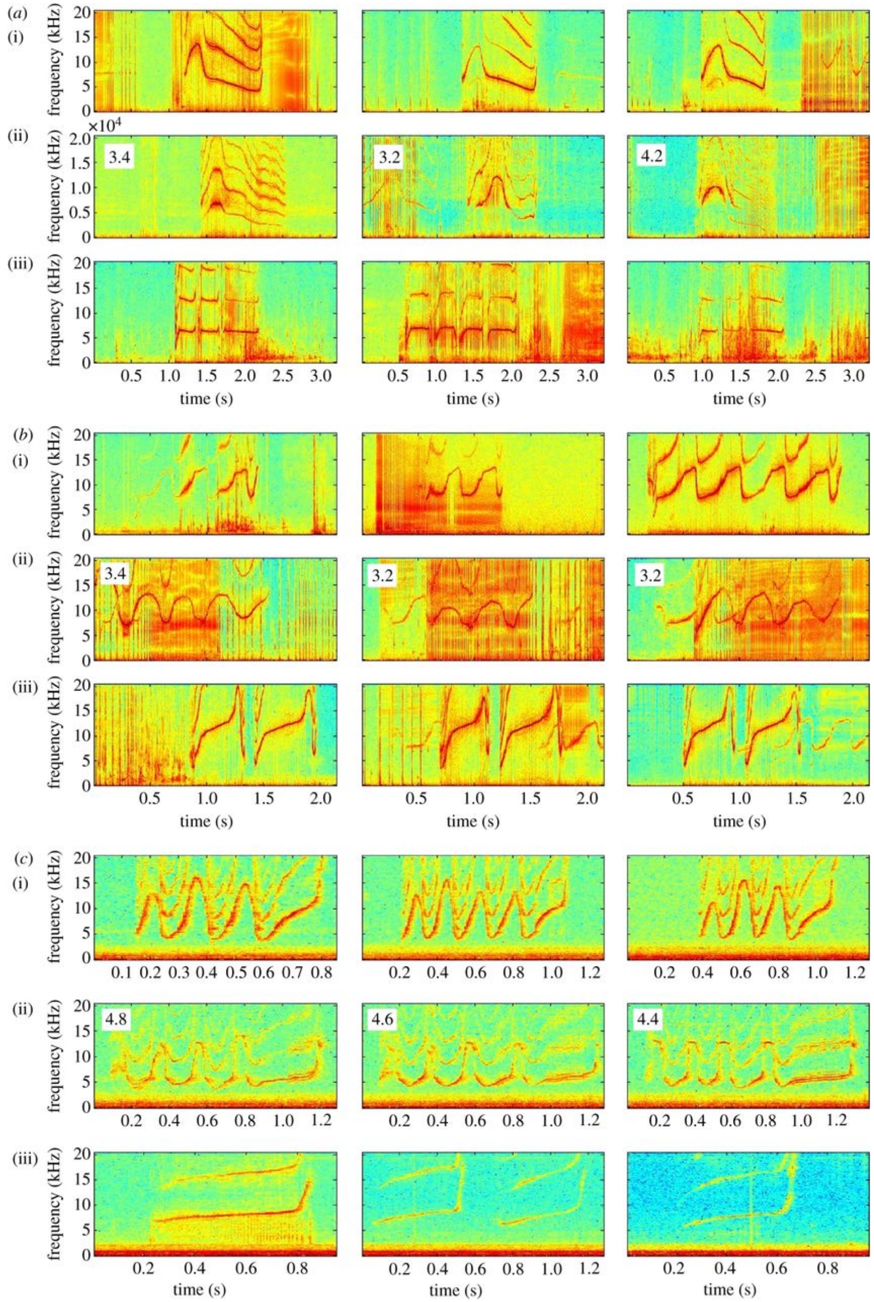
signal example: speech



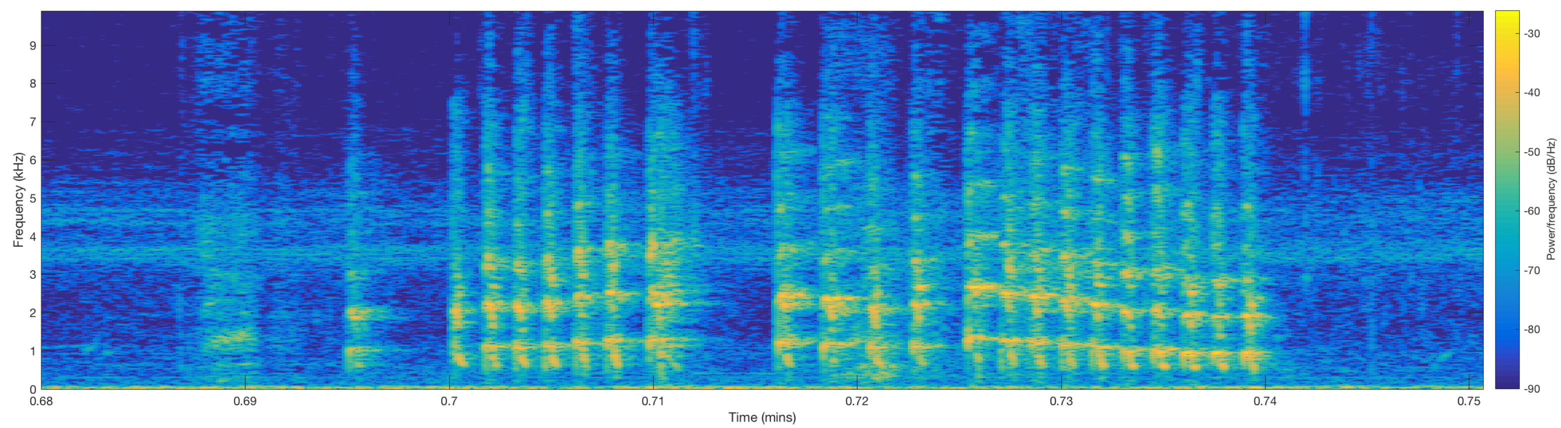
signal example: speech

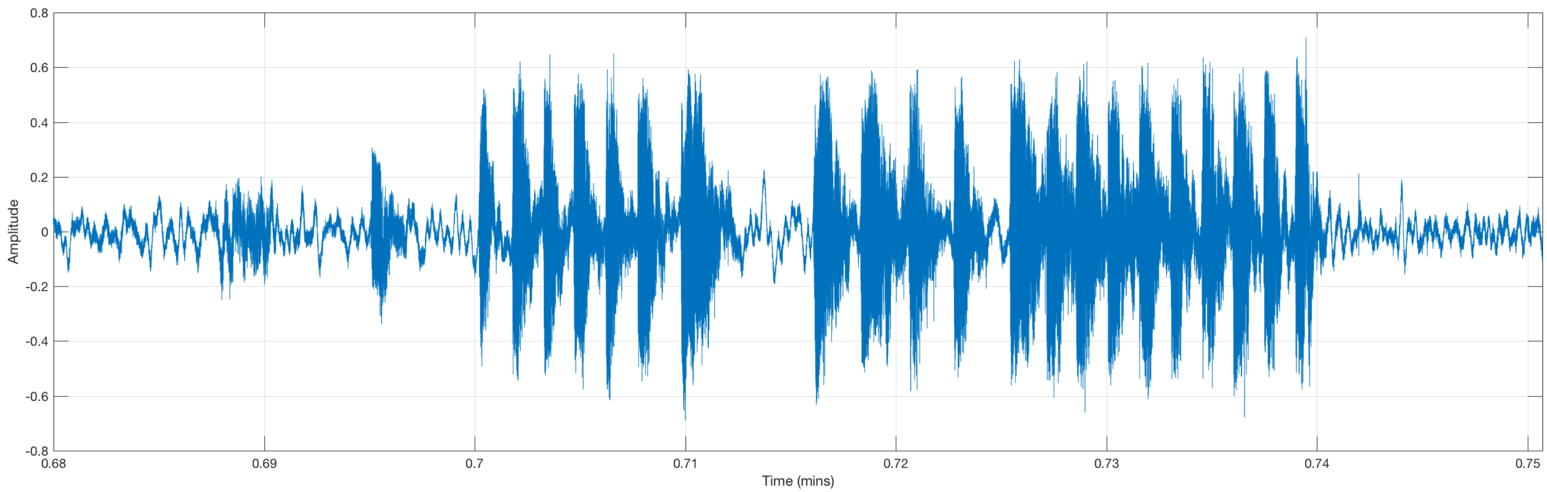


signal example: dolphin

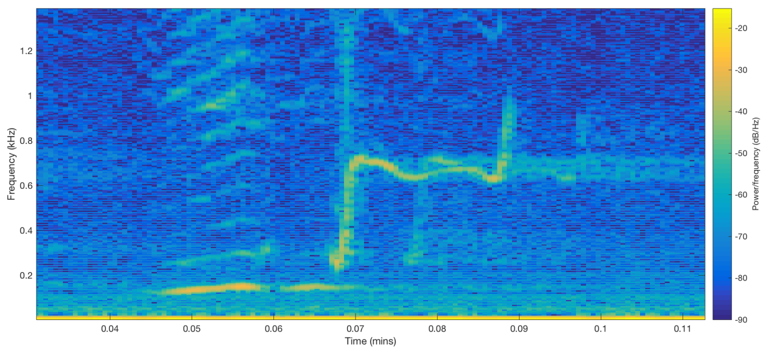


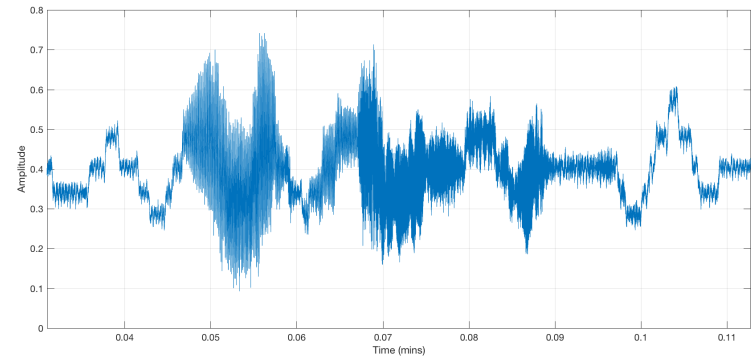
signal example: dolphin

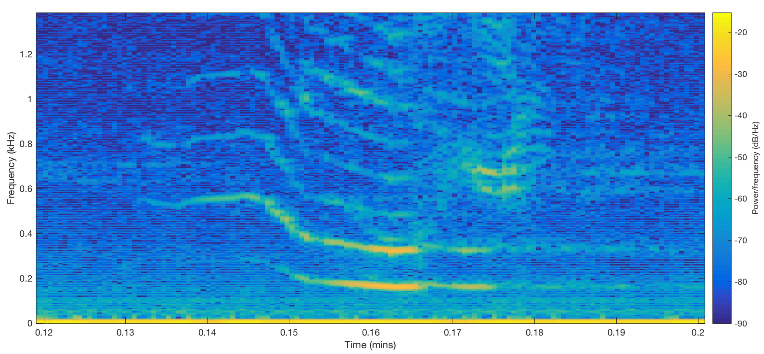


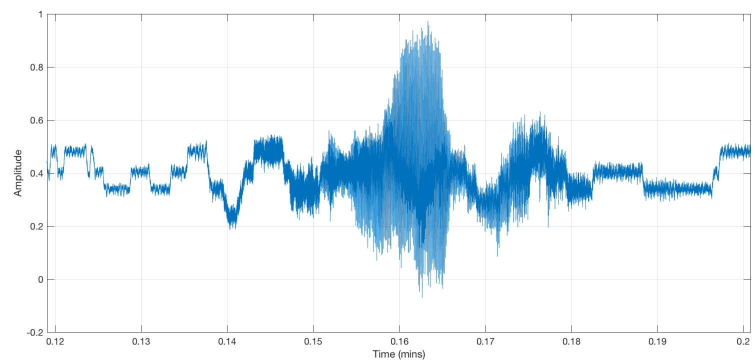


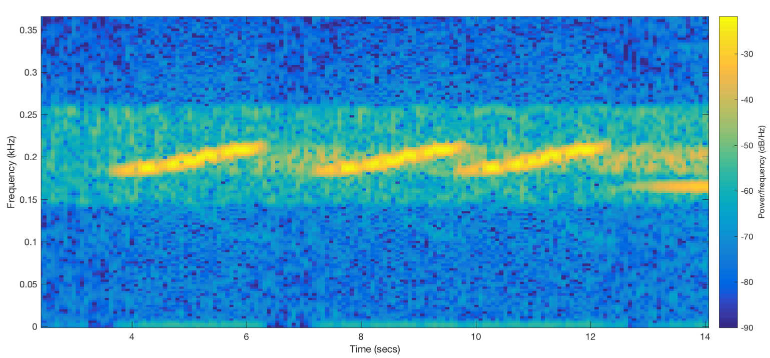
signal example: humpback whale & low-frequency sonar

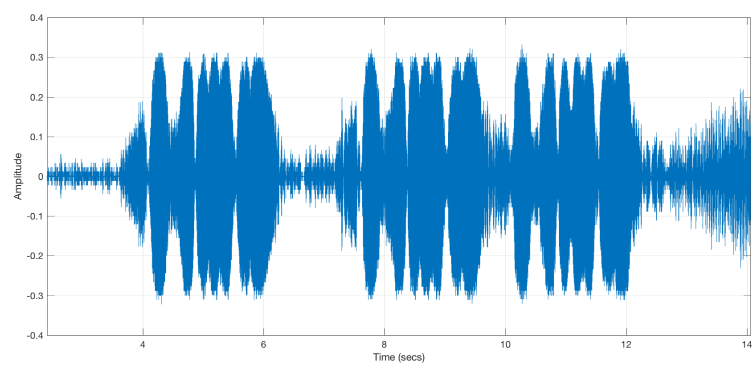




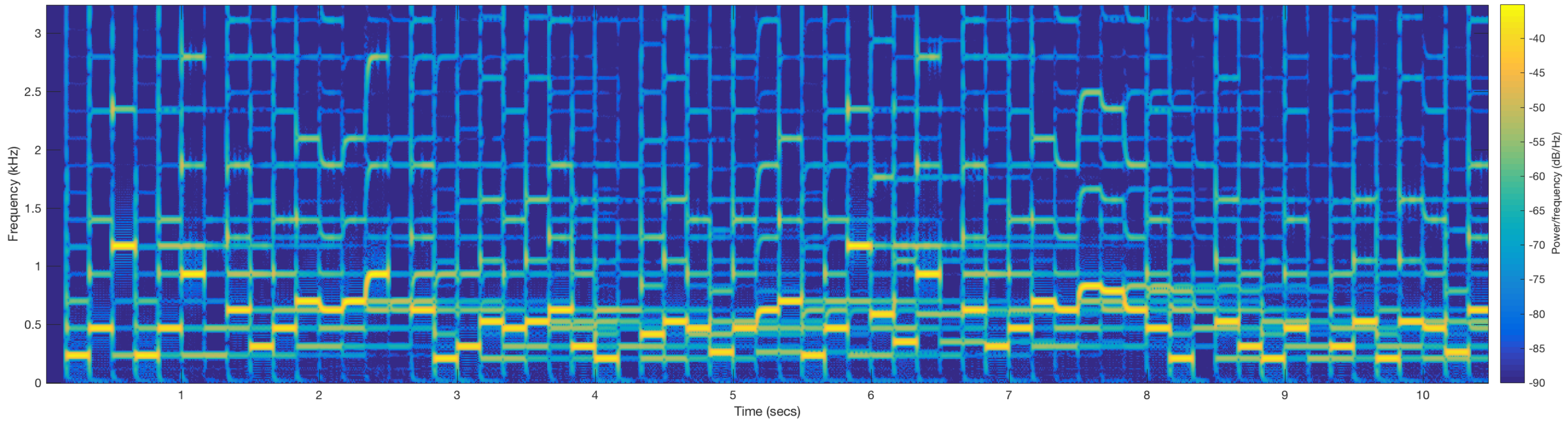


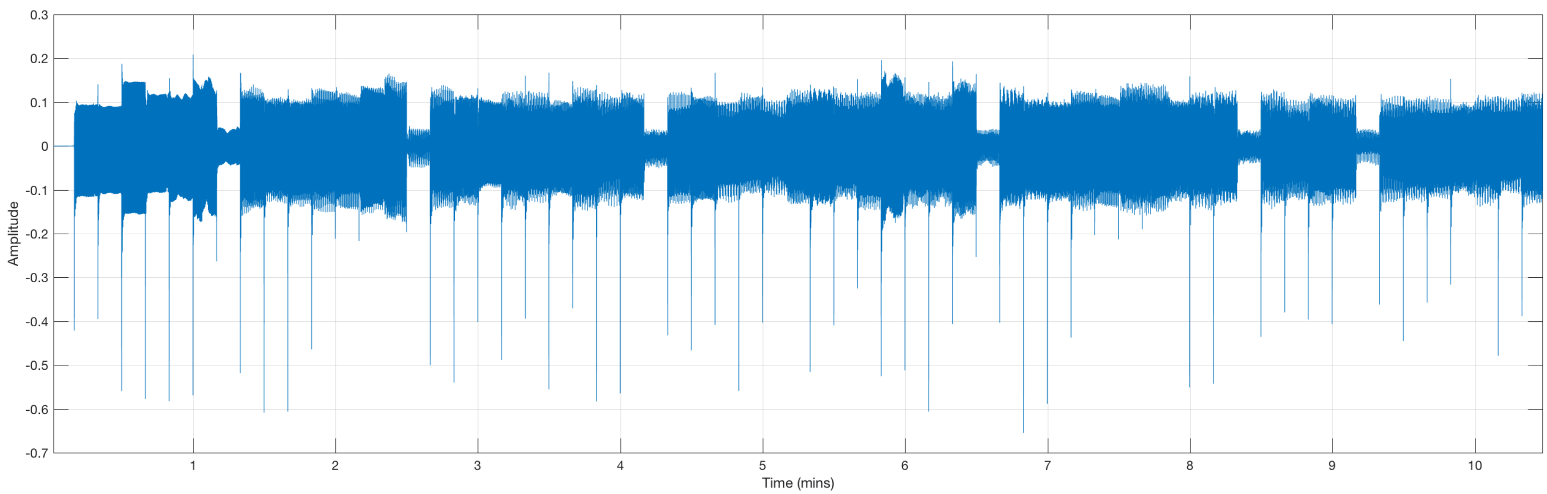






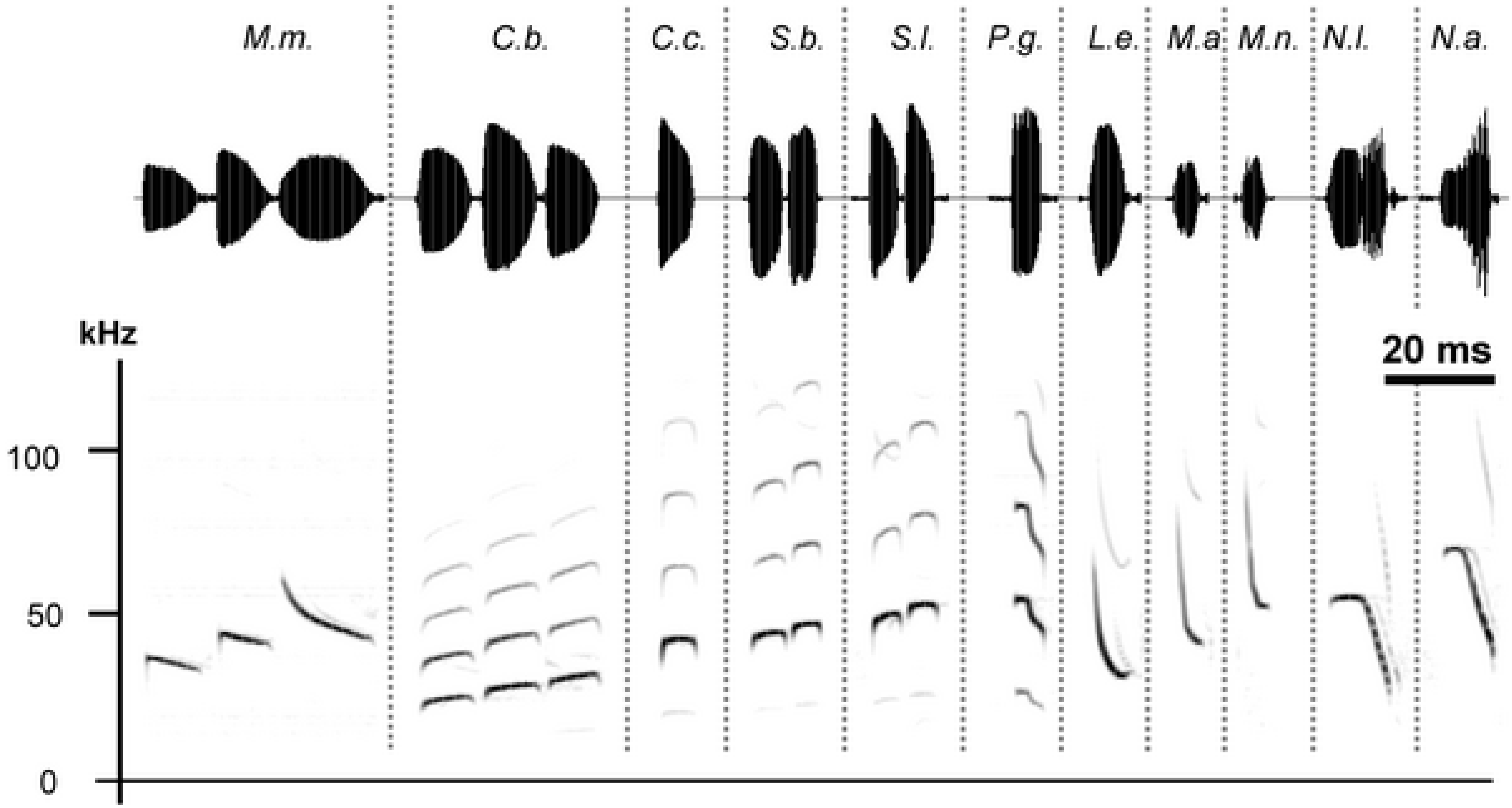
signal example: music (fireflies)





signal example: bat

- How do patterns convey information?
- What are the signal attributes?



signal example: radio waves

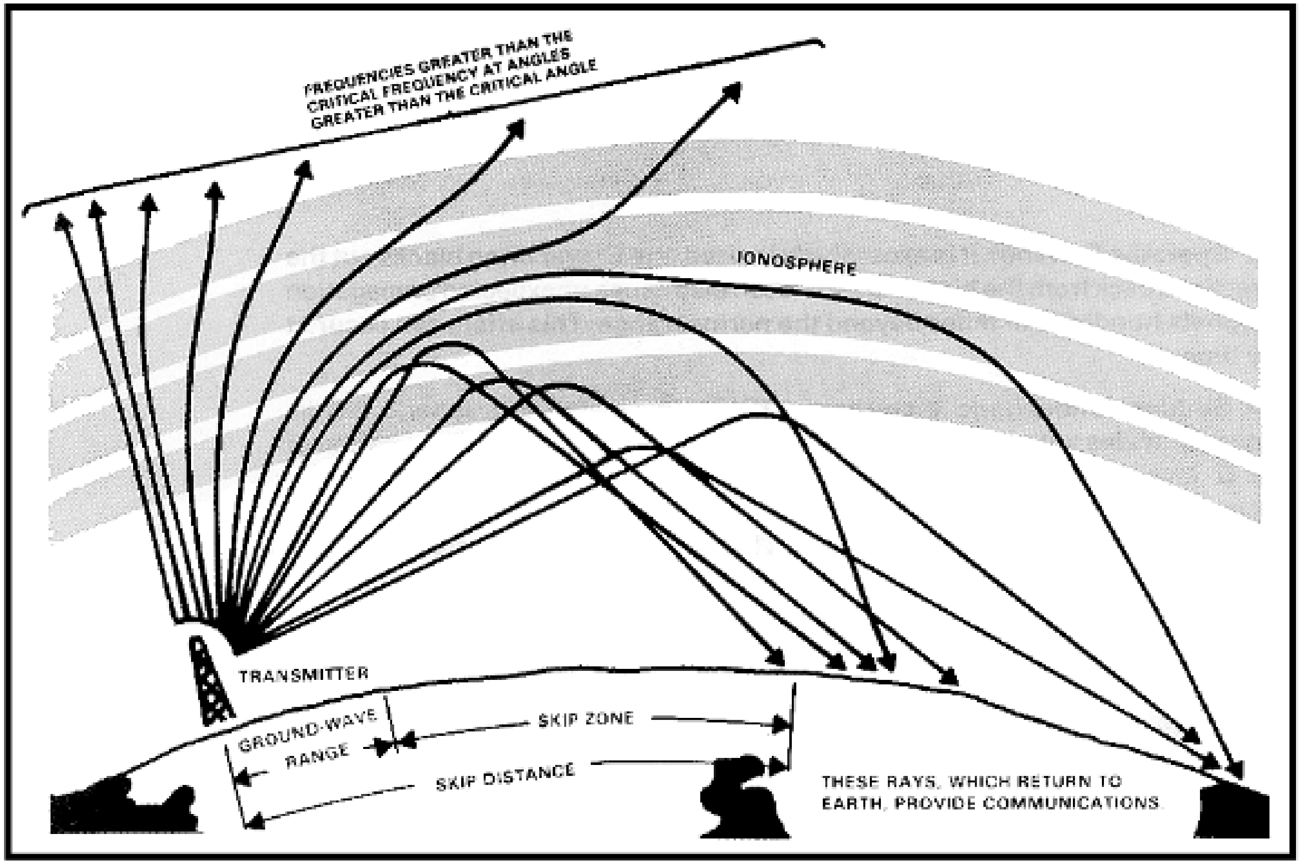
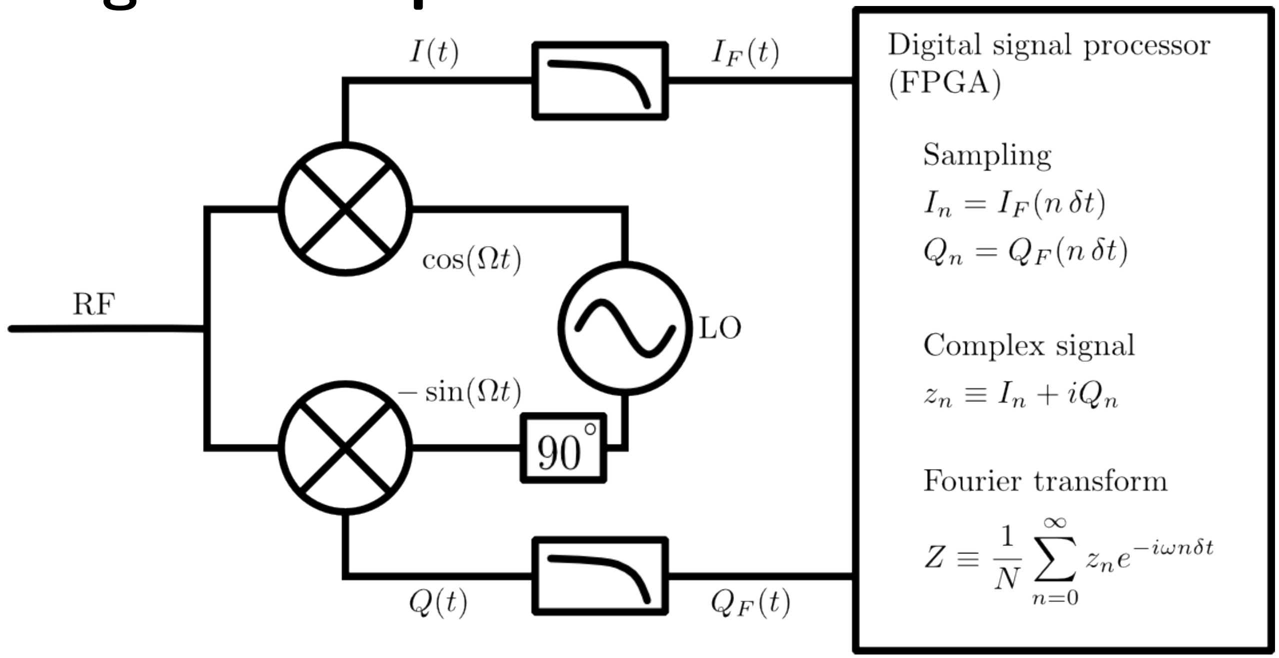
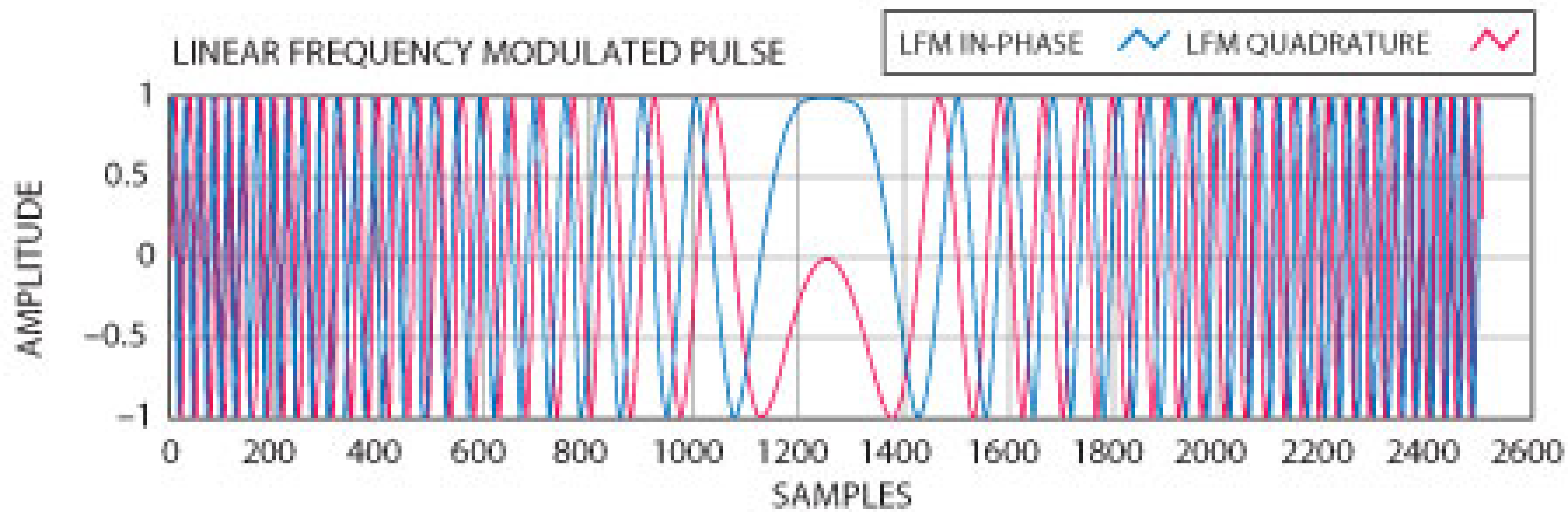


Figure 2-14. Sky wave transmission paths.

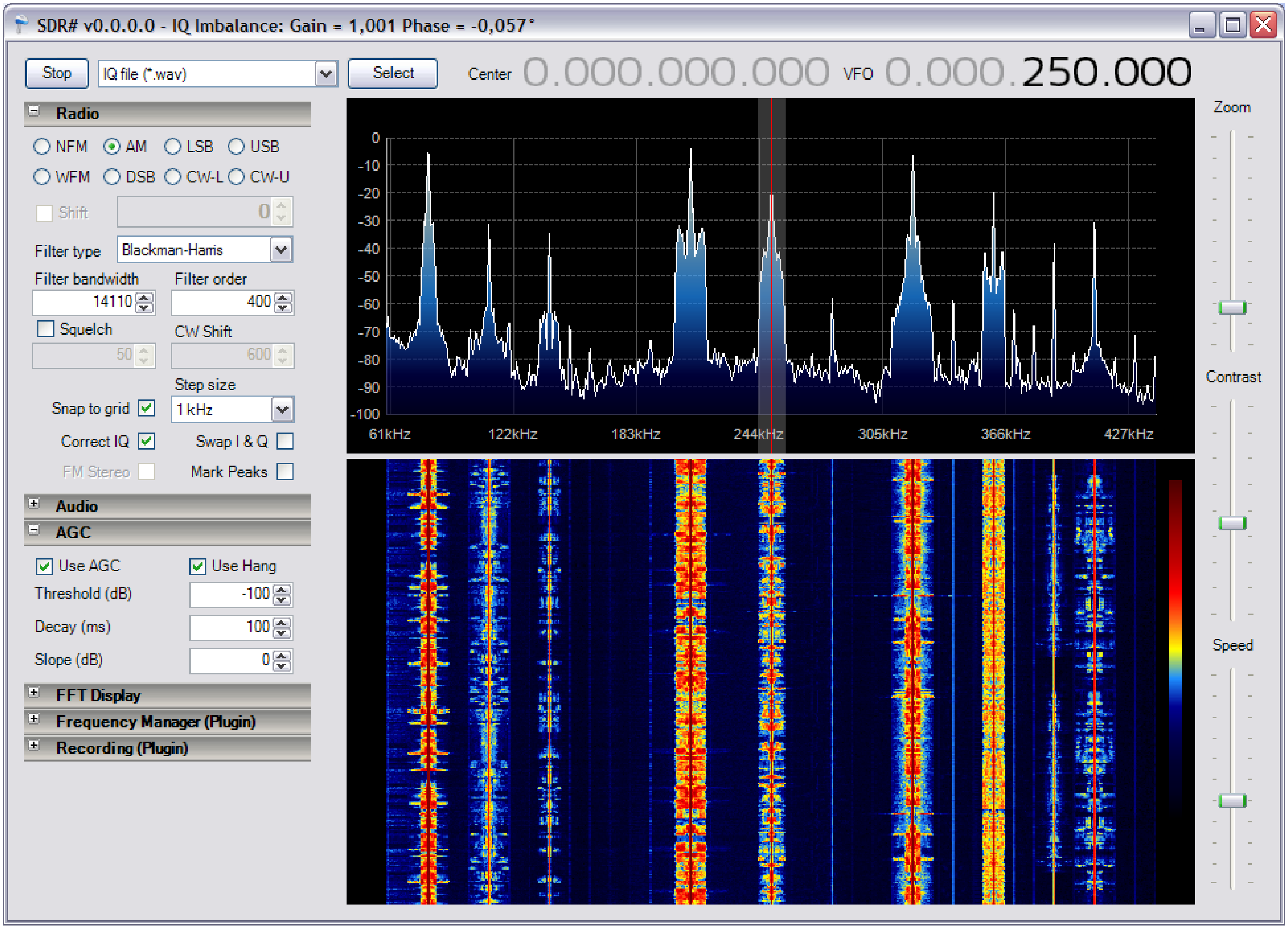
signal example: radar

- How do patterns convey information?
- What are the signal attributes?



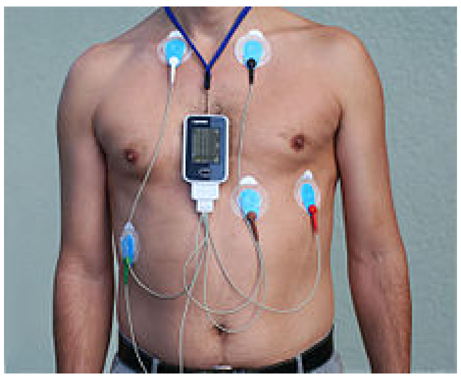


signal example: radio signals



signal example: ECG

- How do patterns convey information?
- What are the signal attributes?



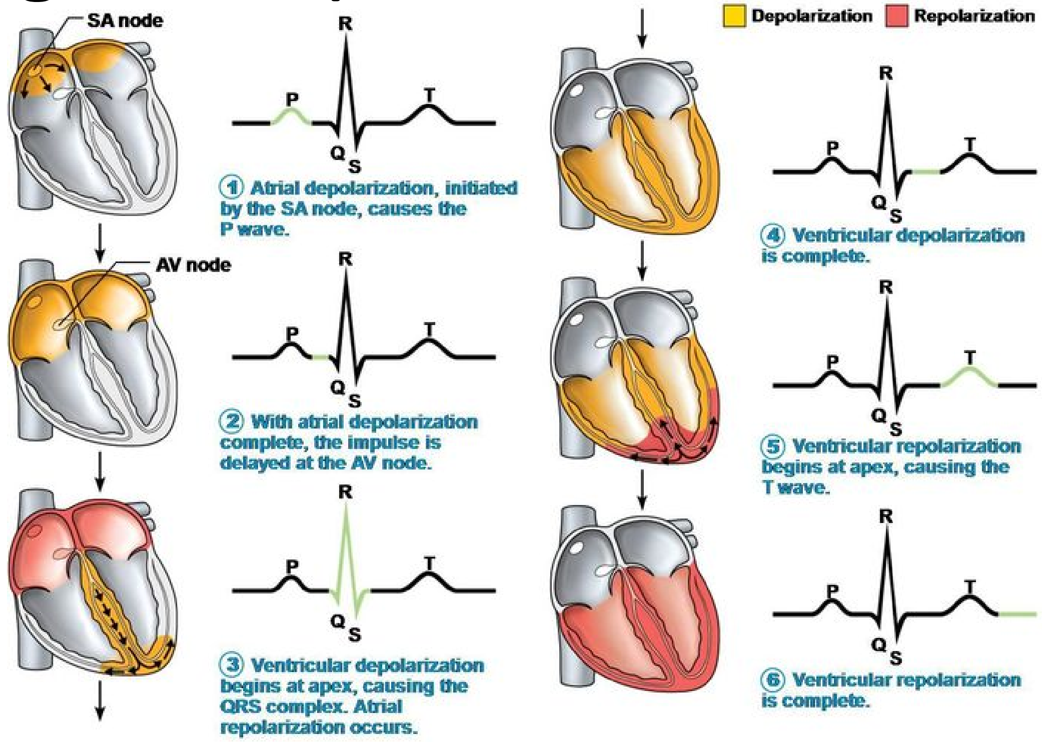
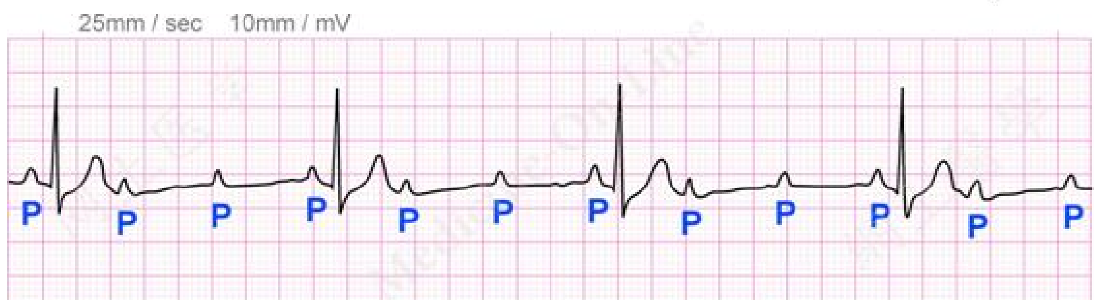
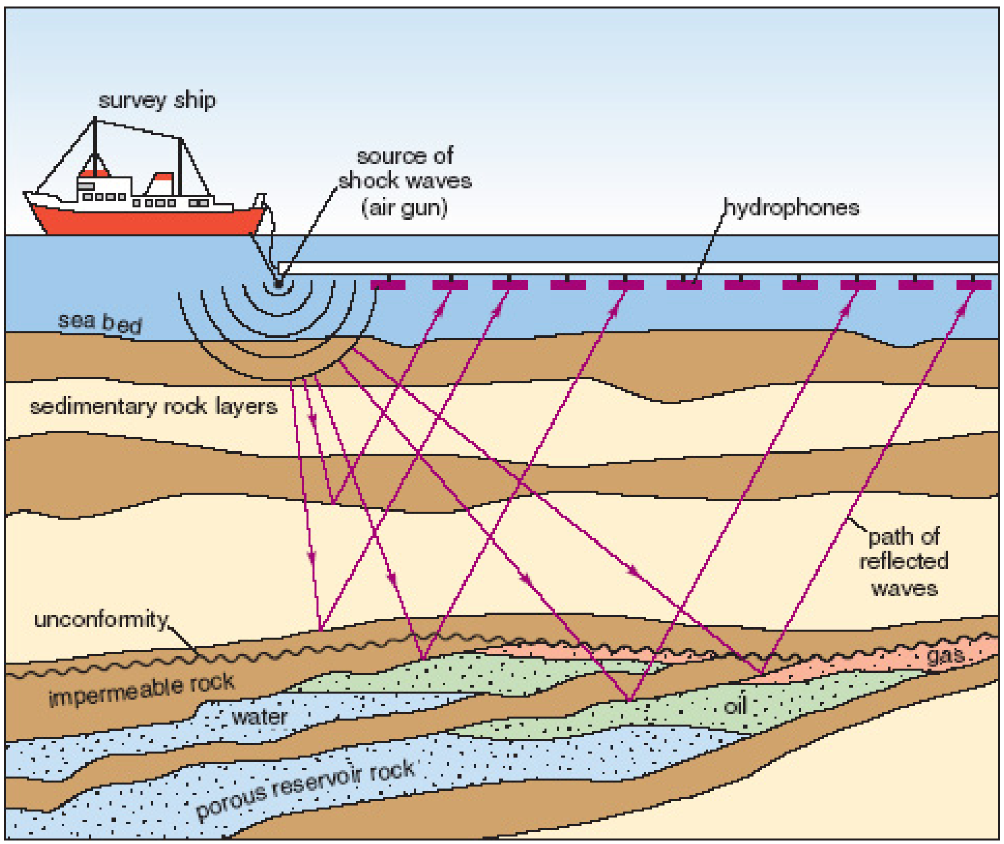


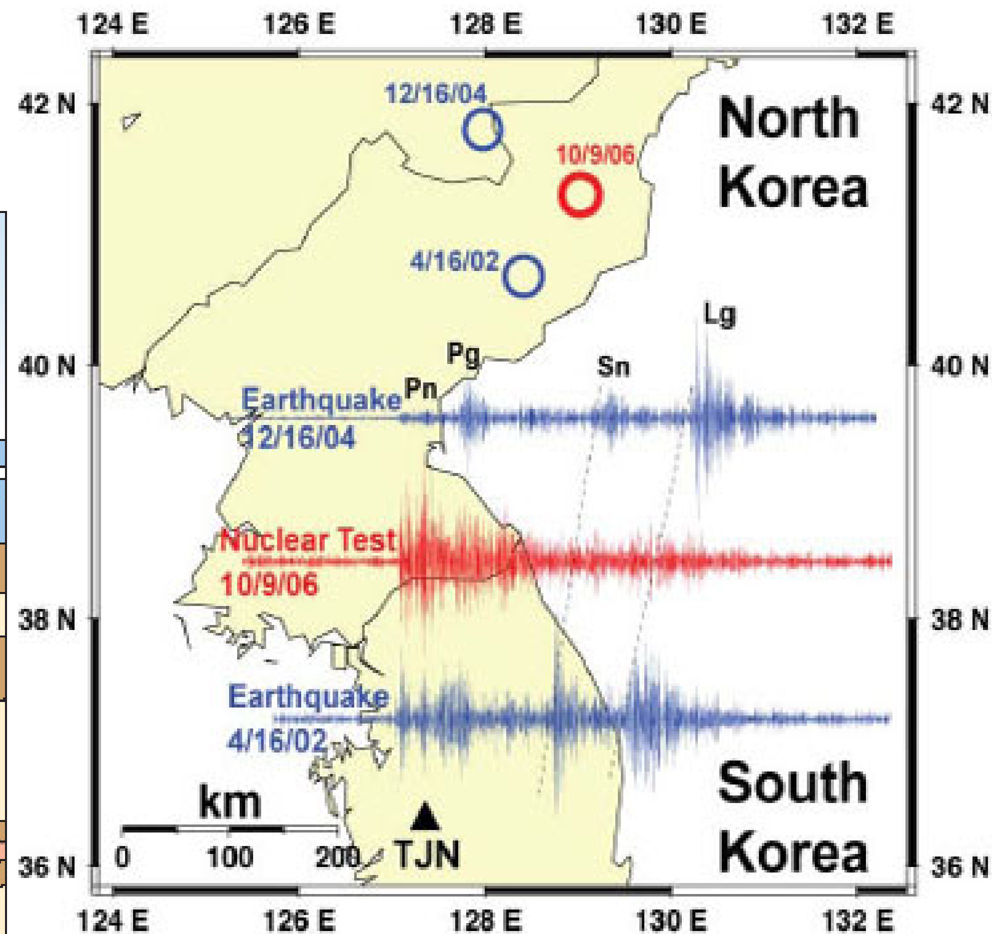
Figure 18.17



signal example: seismic exploration

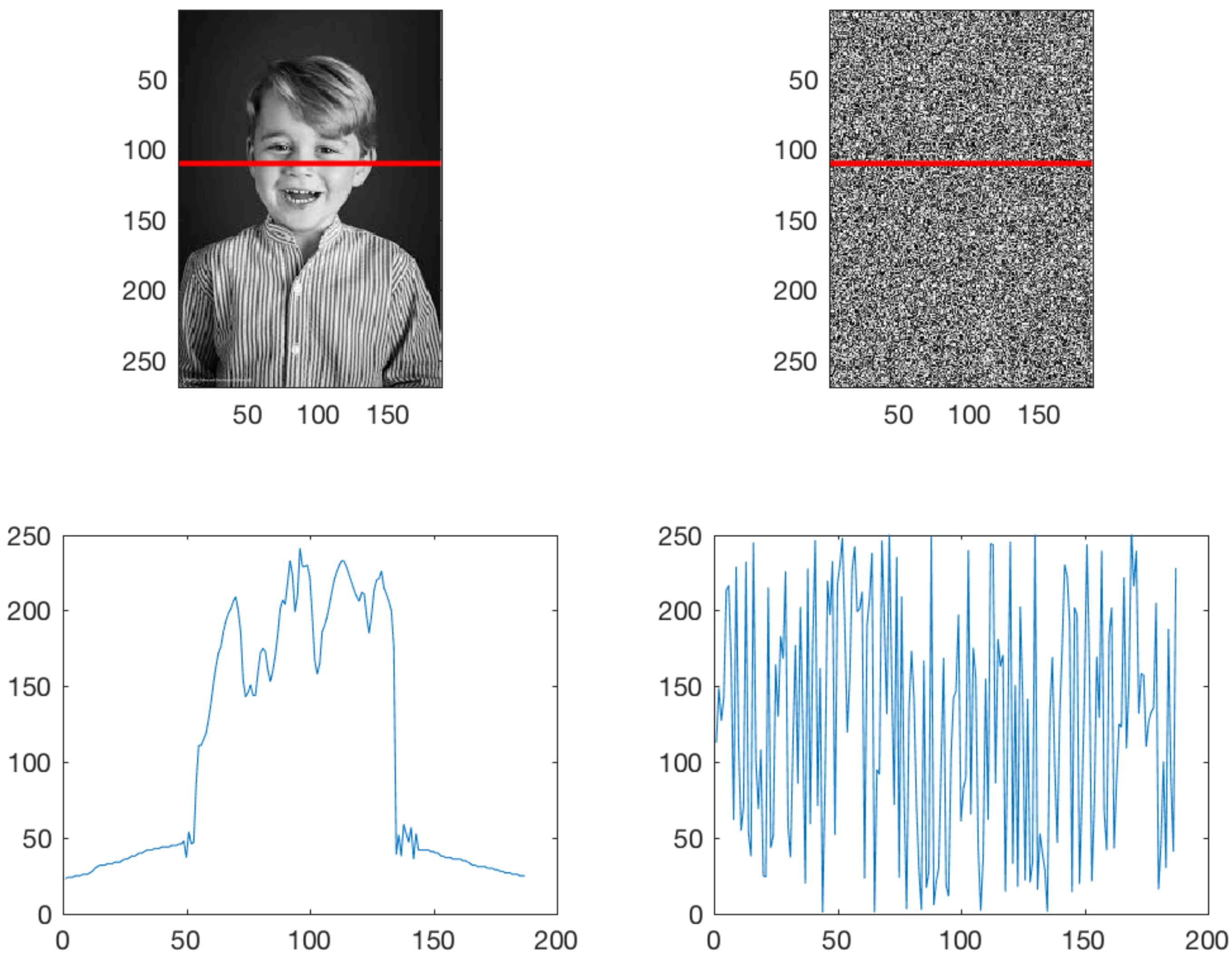
- How do patterns convey information?
- What are the signal attributes?





signal example: image

- How do patterns convey information?
- What are the signal attributes?

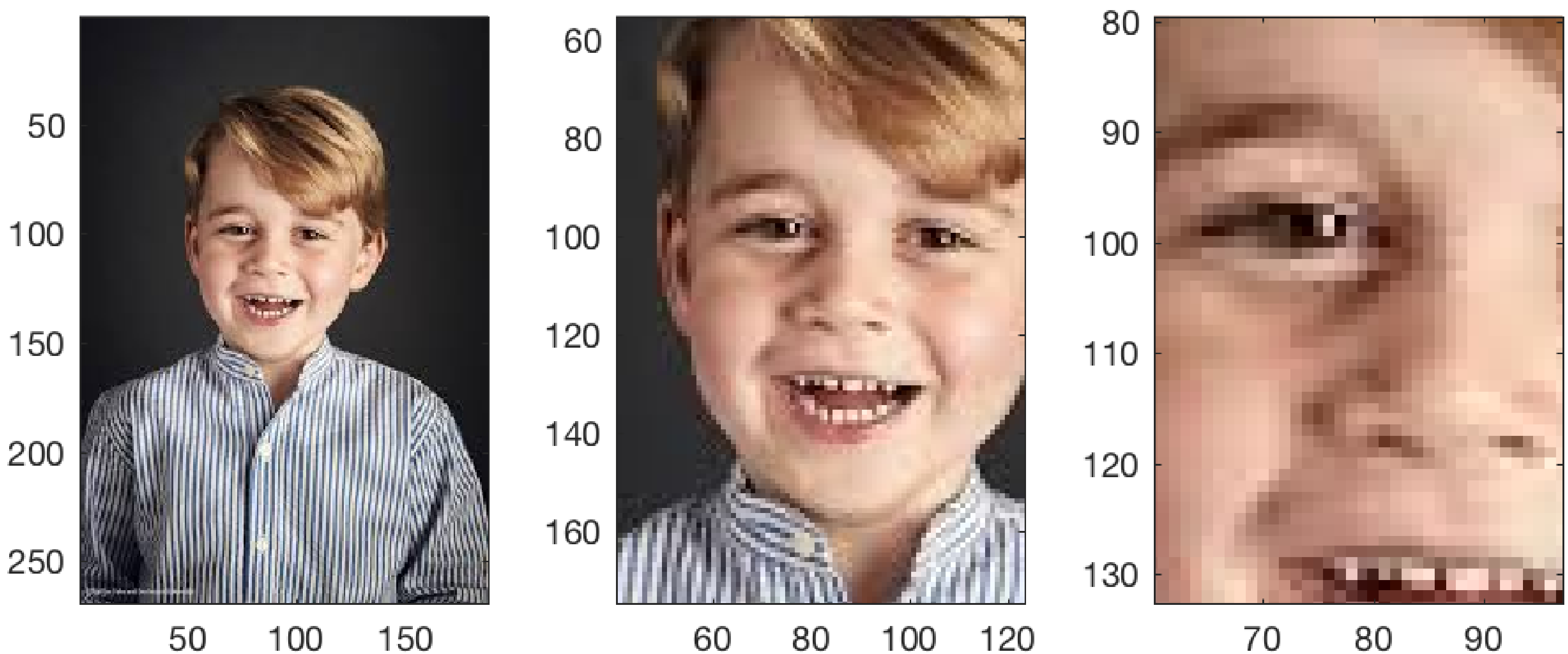


signal example: image

- How do patterns convey information?
- What are the signal attributes?

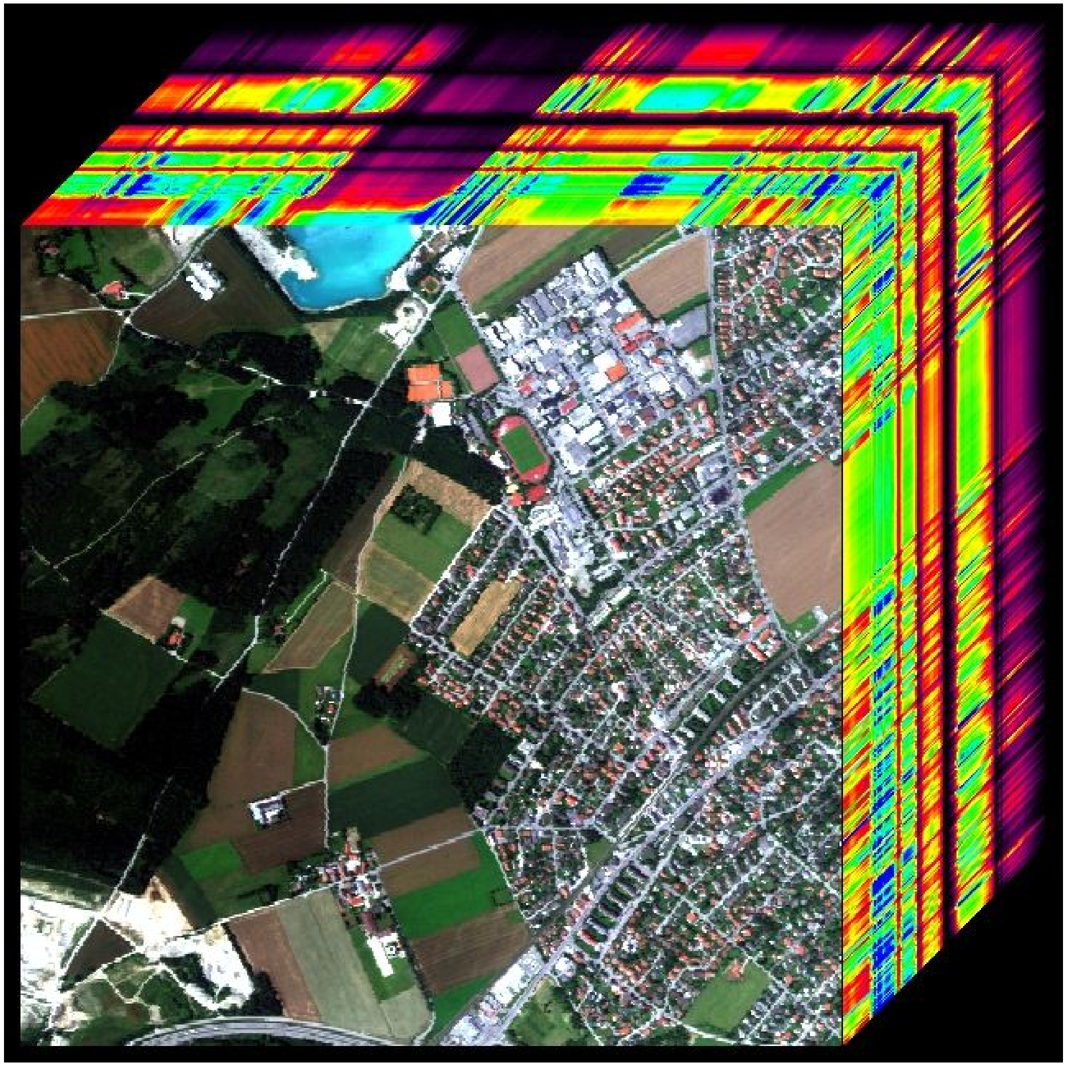
signal example: image

- How do patterns convey information?
- What are the signal attributes?



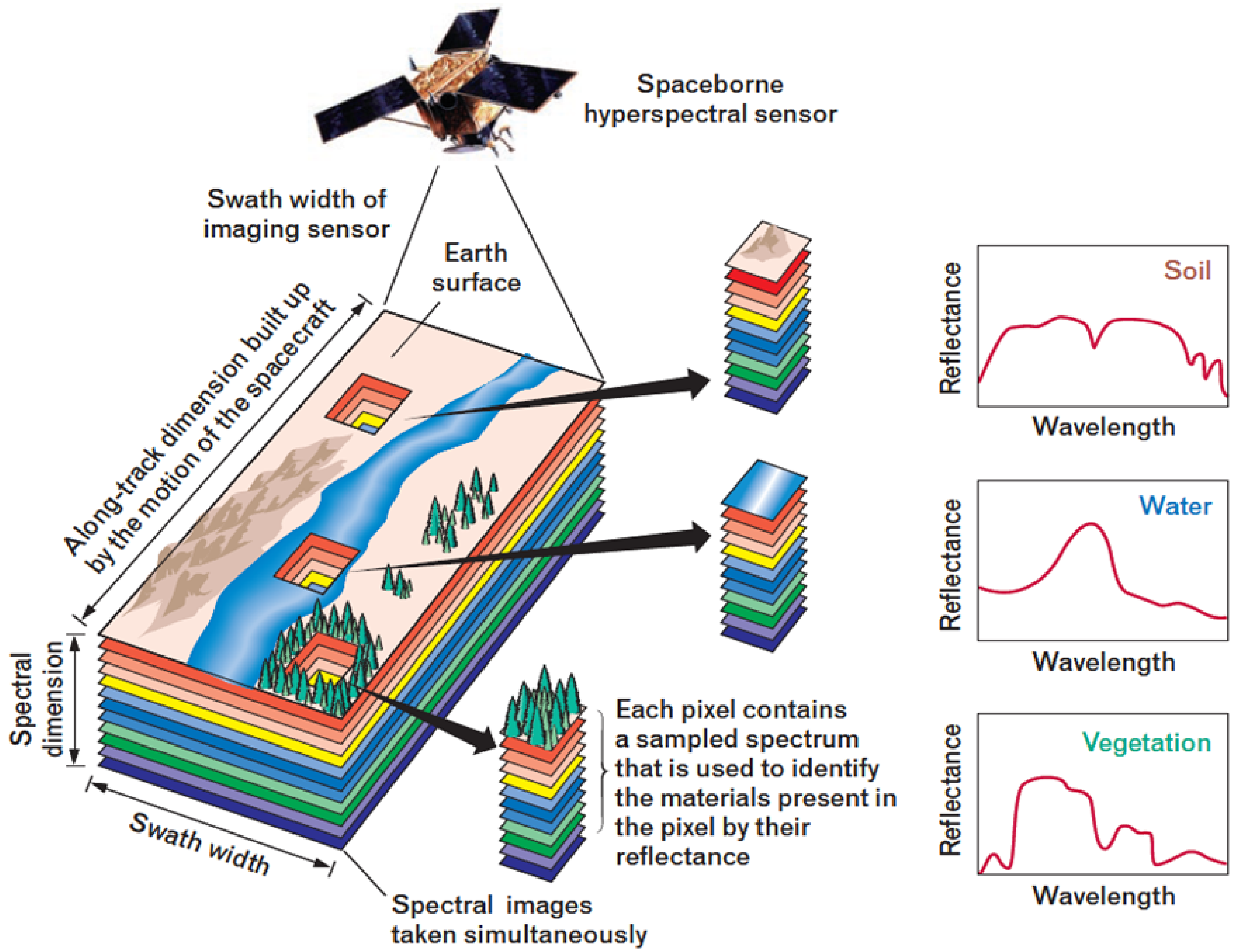
signal example: hyperspectral image

What is the dimension and number of channels in a hyperspectral image?



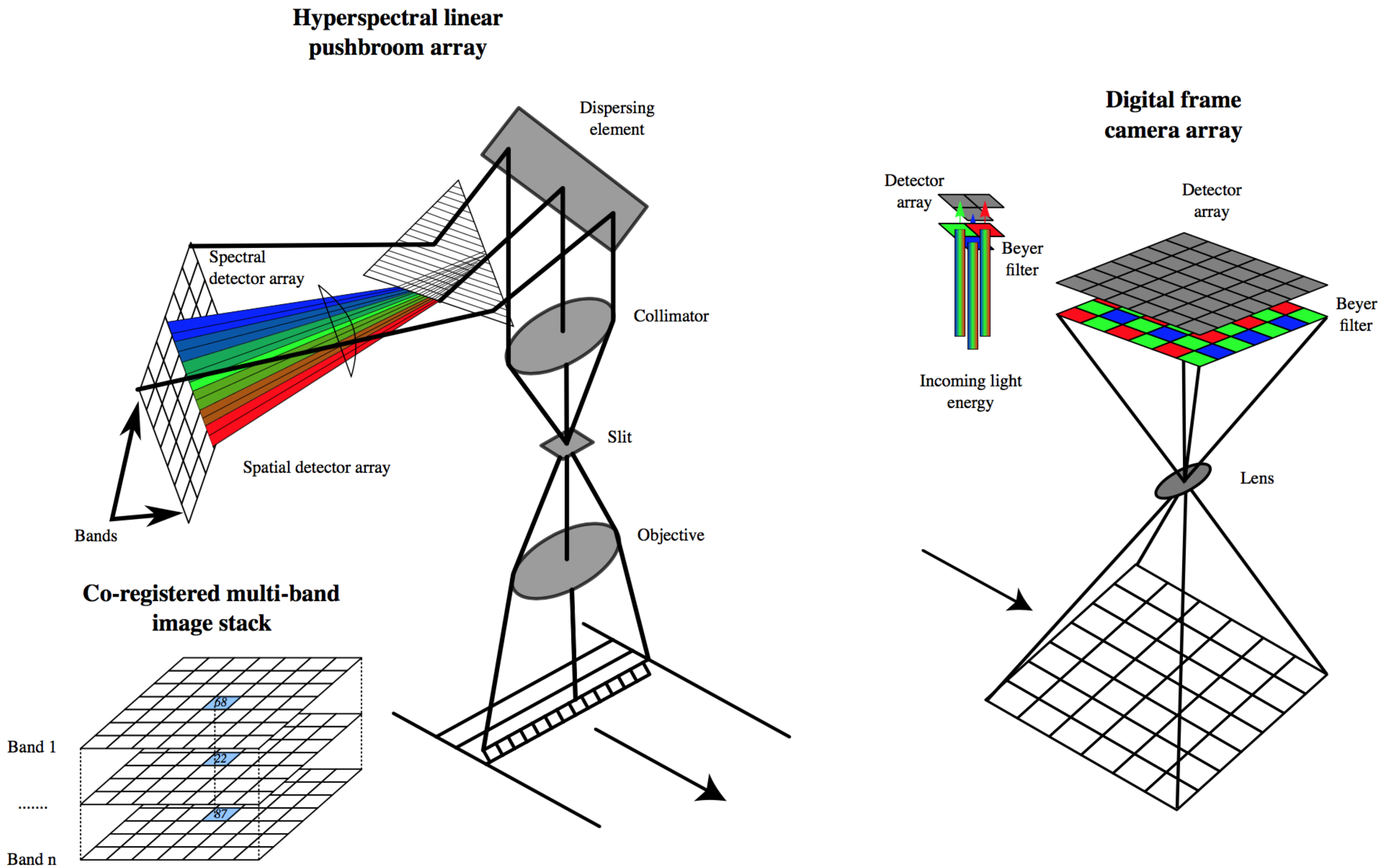
hyperspectral image

What is the dimension and number of channels in a hyperspectral image?



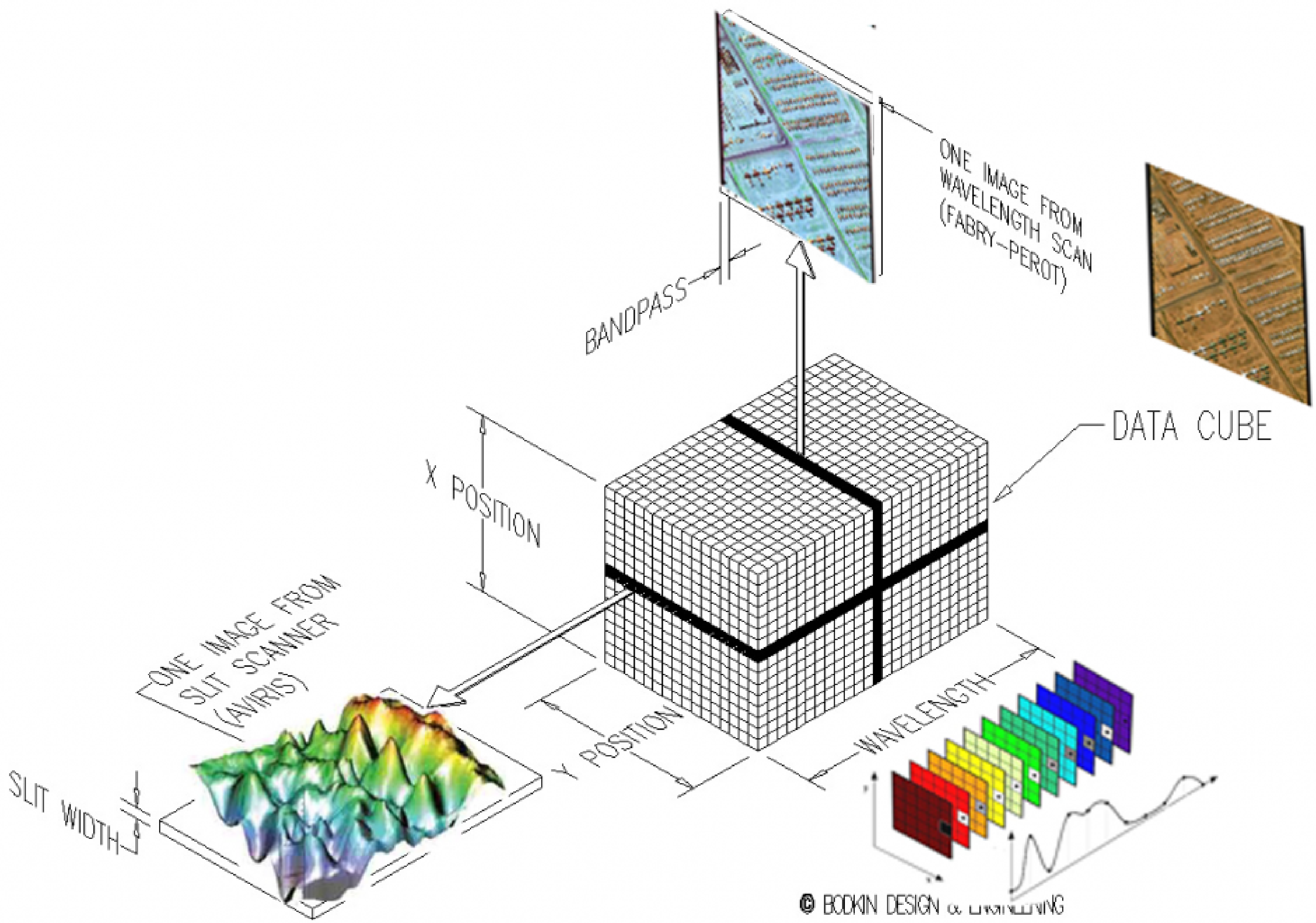
hyperspectral image

What is the dimension and number of channels in a hyperspectral image?



hyperspectral image

What are the attributes of a hyperspectral image? (dimension, channels, continuous vs. discrete, analog vs. digital)



daily closing stock price

What are the attributes of a stock price? (dimension, channels, continuous vs. discrete, analog vs. digital)

musical score

Is this a signal? If so, what are its attributes? (dimension, channels, continuous vs. discrete, analog vs. digital)

text

Is this a signal? If so, what are its attributes? (dimension, channels, continuous vs. discrete, analog vs. digital)

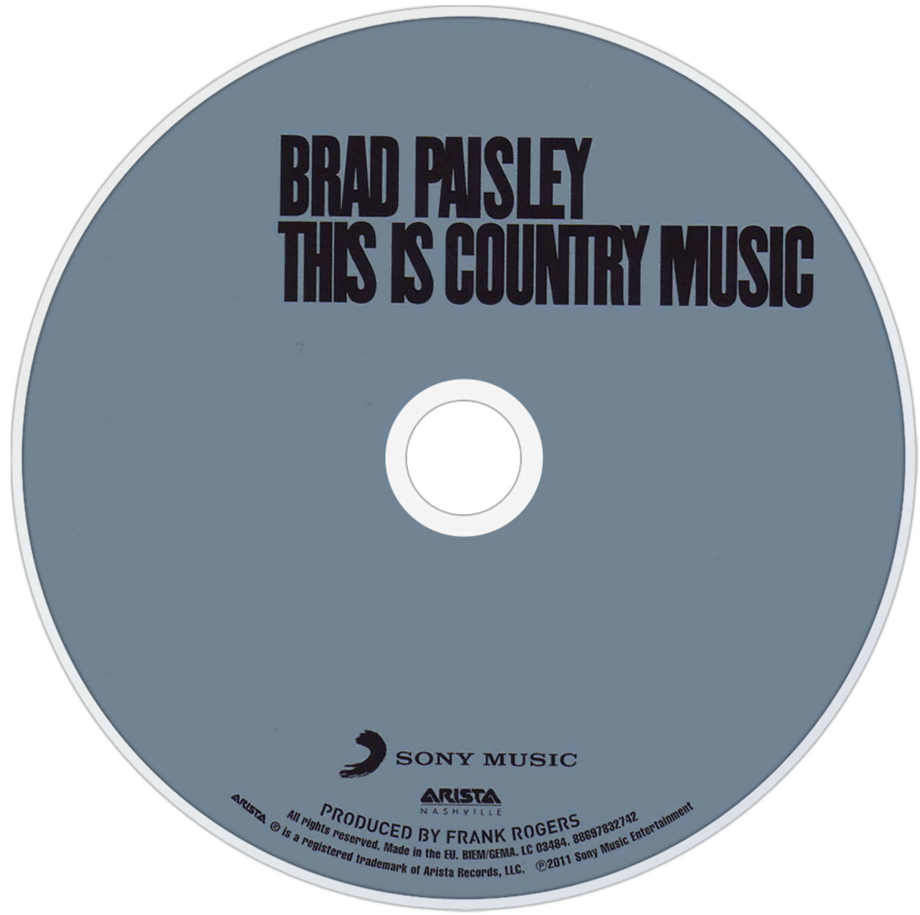
It was the best of times,it was the worst of times, it was the age of wisdom, it was the age of foolishness, it was the epoch of belief, it was the epoch of incredulity, it was the season of light, it was the season of darkness, it was the spring of hope, it was the winter of despair, we had everything before us, we had nothing before us, we were all going direct to Heaven, we were all going direct the other way— in short, the period was so far like the present period, that some of its noisiest authorities insisted on its being received, for good or for evil, in the superlative degree of comparison only.

There were a king with a large jaw and a queen with a plain face, on the throne of England; there were a king with a large jaw and a queen with a fair face, on the throne of France. In both countries it was clearer than crystal to the lords of the State preserves of loaves and fishes, that things in general were settled for ever.

It was the year of Our Lord one thousand seven hundred and seventy-five. Spiritual revelations were conceded to England at that favoured period, as at this. Mrs. Southcott had recently attained her five-and-twentieth blessed birthday, of whom a prophetic private in the Life Guards had heralded the sublime appearance by announcing that arrangements were made for the swallowing up of London and Westminster. Even the Cock-lane ghost had been laid only a round dozen of years, after rapping out its messages, as the spirits of this very year last past (supernaturally deficient in originality) rapped out theirs. Mere messages in the earthly order of events had lately come to the English Crown and People, from a congress of British subjects in America: which, strange to relate, have proved more important to the human race than any communications yet received through any of the chickens of the Cock-lane brood.

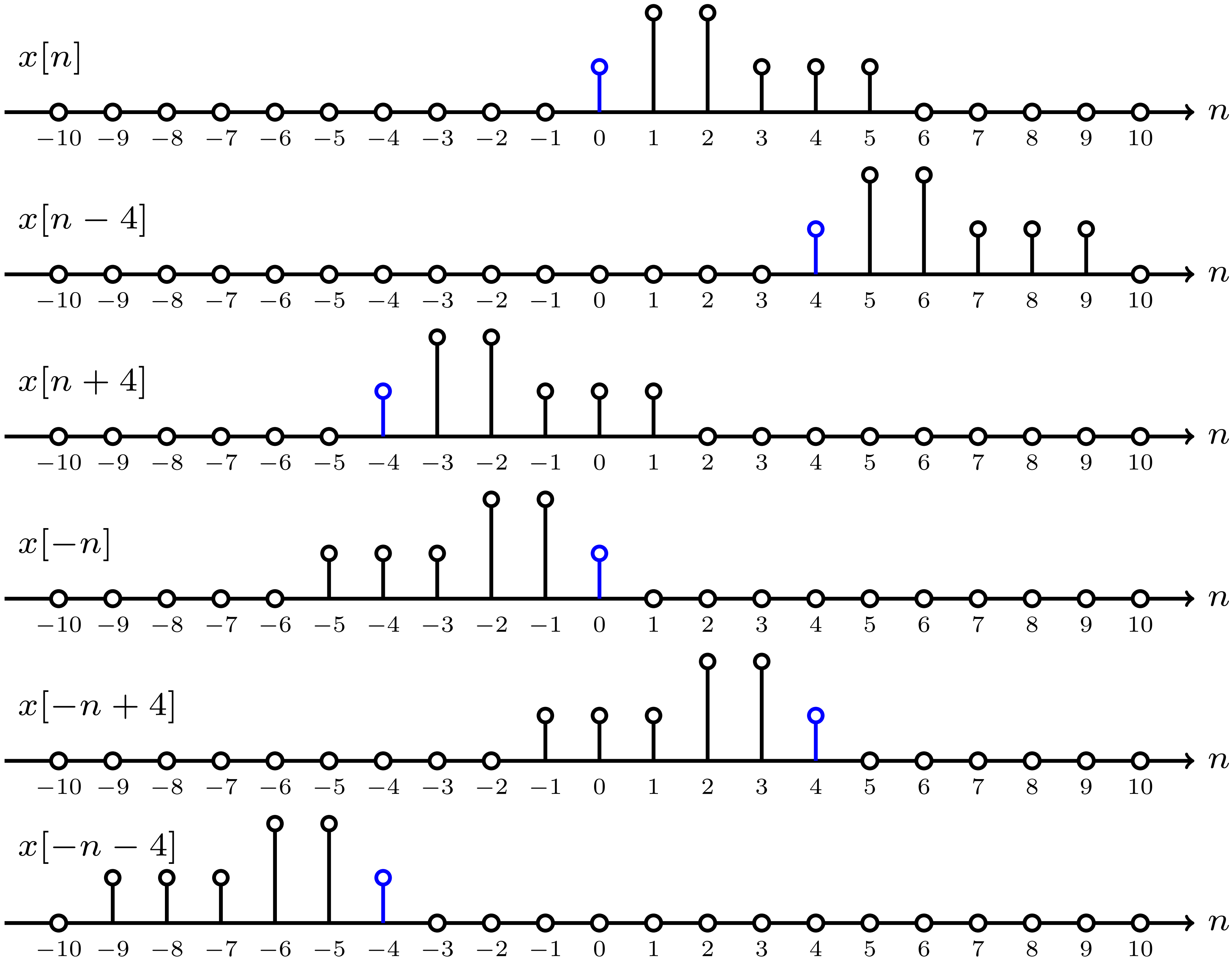
recorded music

Is this a signal? If so, what are its attributes? (dimension, channels, continuous vs. discrete, analog vs. digital)



operations on signals

shifting and reversal



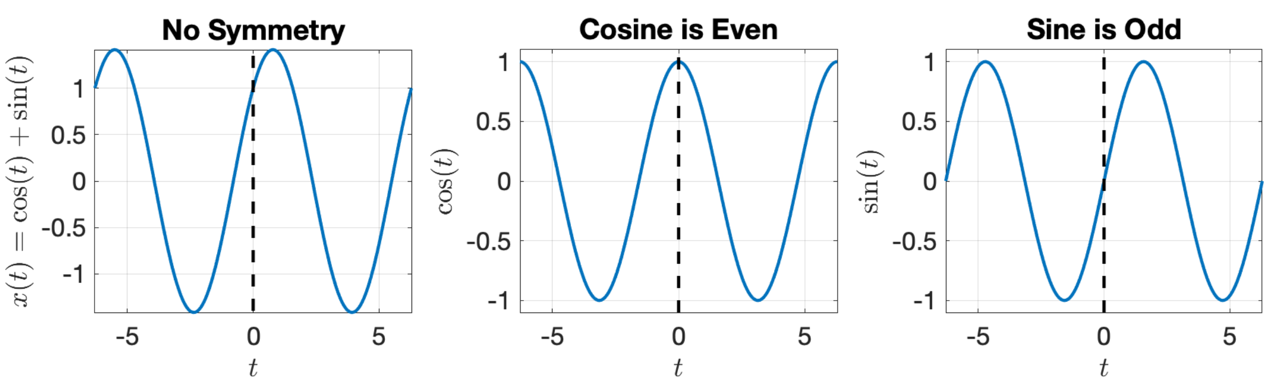
shifting and reversal

Shifting and reversal are used in convolution:

$$y[n] = x[n] * h[n] = \sum_k x[k]h[n-k]$$

Shifting and reversal are used to extract even and odd parts:

$$\begin{aligned} x_{\text{even}}[n] &= \frac{1}{2}\left(x[n] + x[-n]\right), \qquad & x_{\text{even}}[-n] &= x_{\text{even}}[n] \\ x_{\text{odd}}[n] &= \frac{1}{2}\left(x[n] - x[-n]\right), \qquad & x_{\text{odd}}[-n] &= -x_{\text{odd}}[n] \\ x[n] &= x_{\text{even}}[n] + x_{\text{odd}}[n] \end{aligned}$$



common signals

uses of impulse function

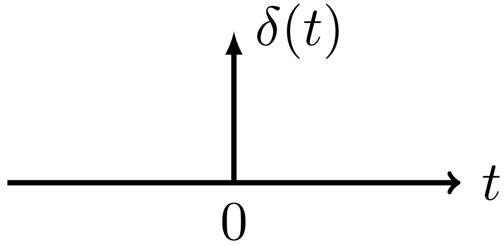
$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

applications:

1. sifting or sampling: $x[n] \cdot \delta[n-k] = x[k]\delta[n-k]$ (multiplication)
2. delay/shift: $x[n] * \delta[n-k] = x[n-k]$ (convolution)
3. representation: $x[n] = x[n] * \delta[n] = \sum_k x[k]\delta[n-k]$
4. periodic construction: $g[n] * \sum_k \delta[n-kN] = \sum_k g[n-kN]$

impulse functions and properties

Dirac delta function

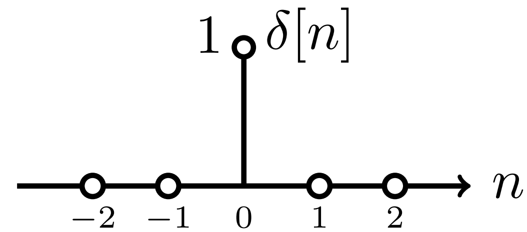


$$\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(\tau) d\tau = 1$$

$$\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau) d\tau = x(t)$$

Kronecker delta function



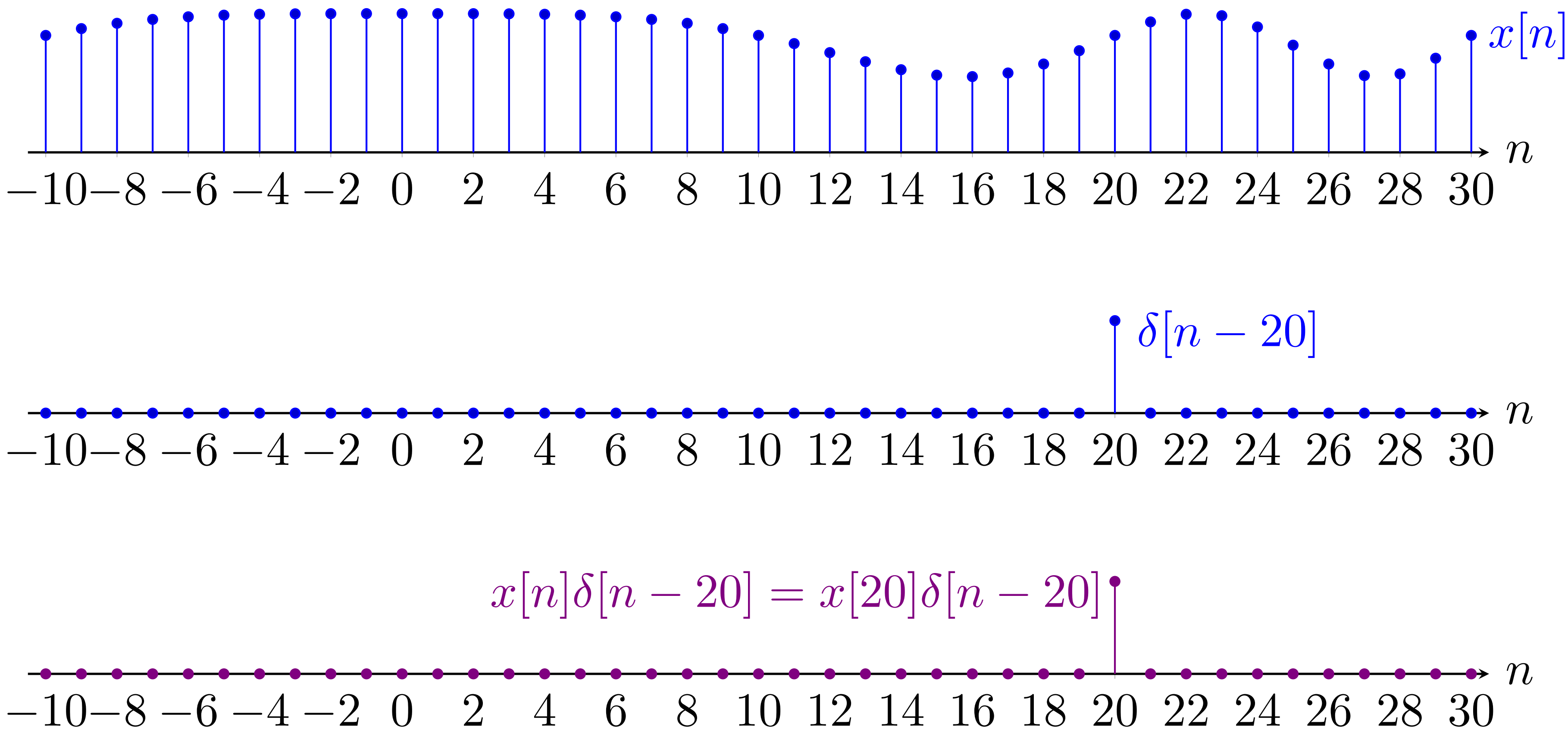
$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

$$\sum_{k=-\infty}^{\infty} \delta[k] = 1$$

$$\sum_{k=-\infty}^{\infty} x[k]\delta[n-k] = x[n]$$

sifting property of Kronecker delta

$$\boxed{x[n]\delta[n-k] = x[k]\delta[n-k]}$$



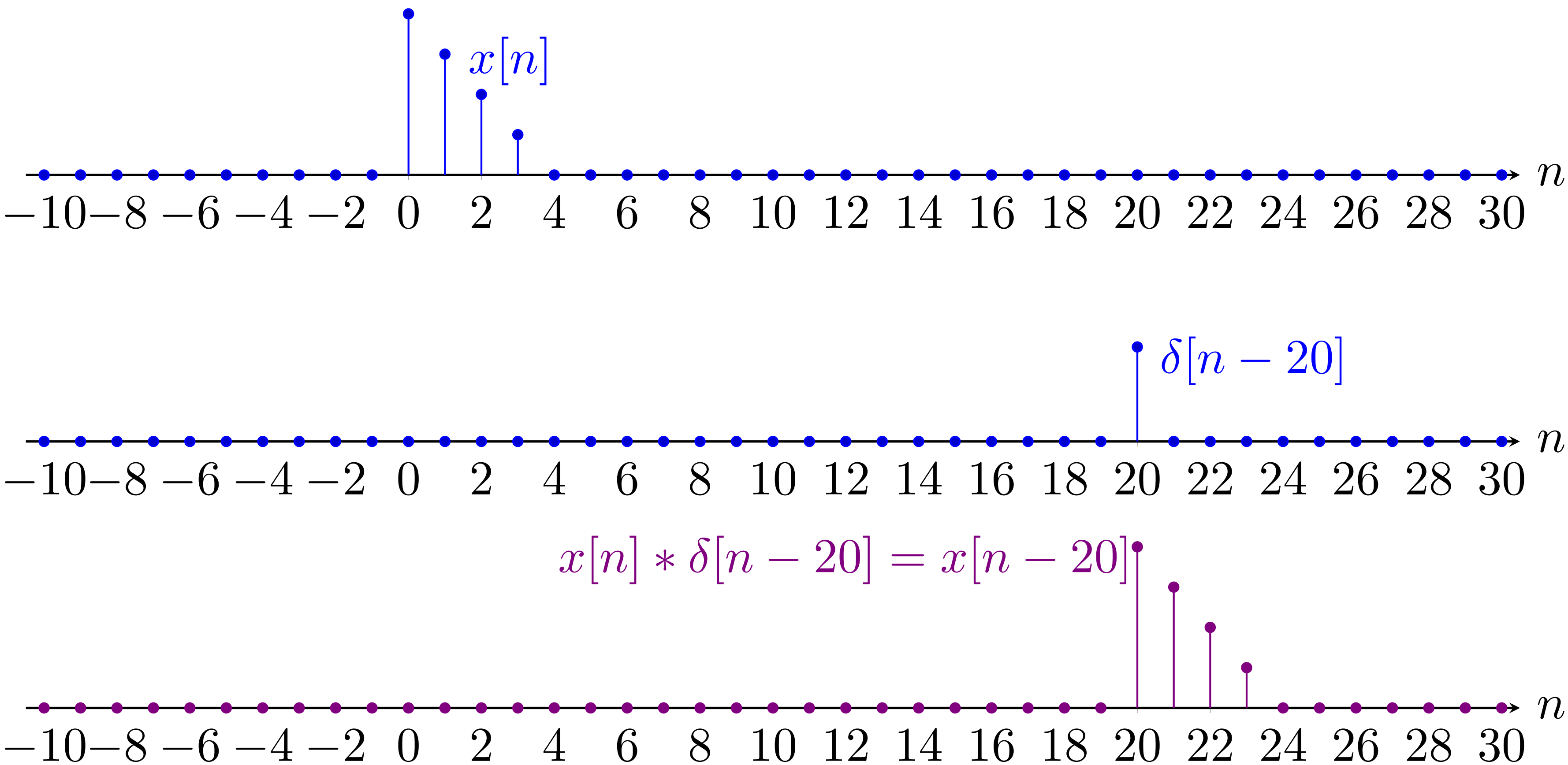
convolution property of Kronecker delta

$$\boxed{x[n] * \delta[n-k] = x[n-k]}$$

$$\begin{aligned}
x[n] * \delta[n-k] &= \sum_i x[i]\delta[(n-i)-k] \\
&= \sum_i x[i]\delta[(n-k)-i] \\
&= \sum_i x[n-k]\delta[(n-k)-i] \quad \text{(sift)} \\
&= x[n-k] \quad \cdot \quad \underbrace{\sum_i \delta[n-k-i]}_{=1} \quad \text{(factor out constant)} \\
&= x[n-k]
\end{aligned}$$

convolution property of Kronecker delta

$$\boxed{x[n] * \delta[n-k] = x[n-k]}$$



representation using Kronecker delta

Note that:

$$1 = \sum_k \delta[n-k]$$

Now multiply signal $x[n]$ by 1 and then sift:

$$x[n] = x[n] \cdot 1 = x[n] \underbrace{\left(\sum_k \delta[n-k] \right)}_{1} = \sum_k x[n]\delta[n-k] = \sum_k x[k]\delta[n-k]$$

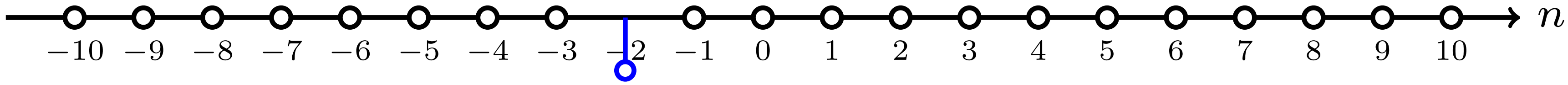
We have:

$$\boxed{x[n] = \sum_k x[k]\delta[n-k]}$$

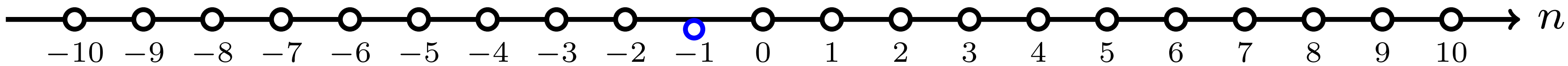
representation using Kronecker delta

$$\vdots$$

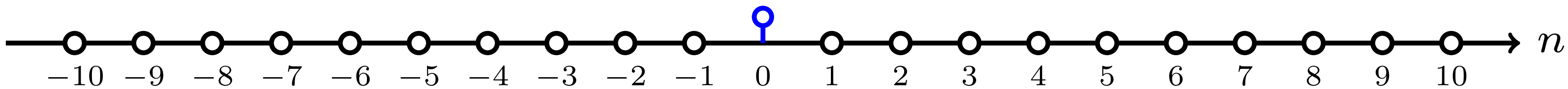
$x[n]\delta[n+2] = x[-2]\delta[n+2]$



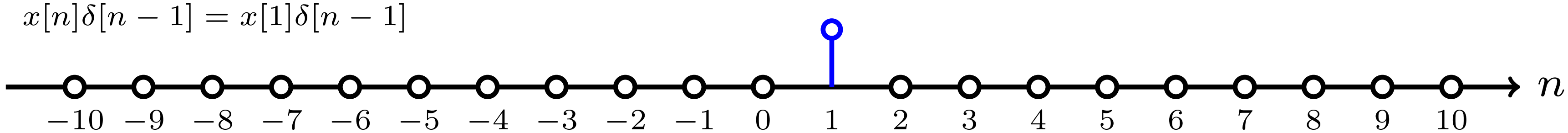
$x[n]\delta[n+1] = x[-1]\delta[n+1]$



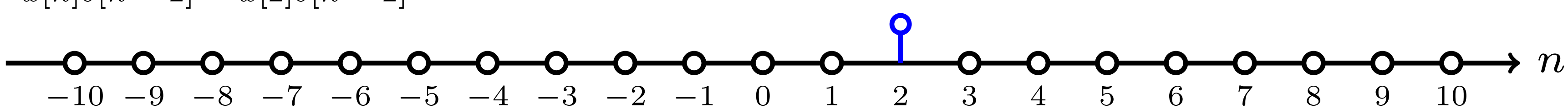
$x[n]\delta[n] = x[0]\delta[n]$



$x[n]\delta[n-1] = x[1]\delta[n-1]$

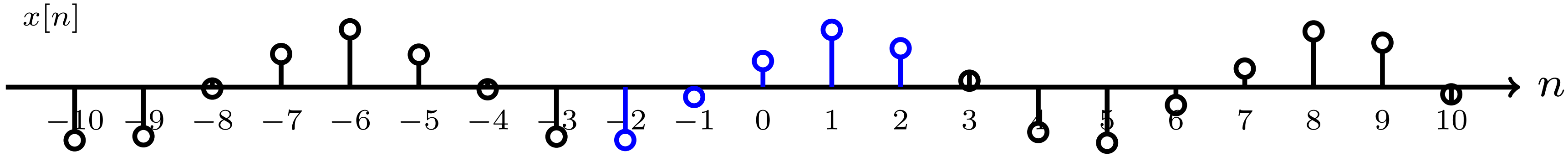


$x[n]\delta[n-2] = x[2]\delta[n-2]$



$$\vdots$$

$x[n]$



impulse train and pulse train construction

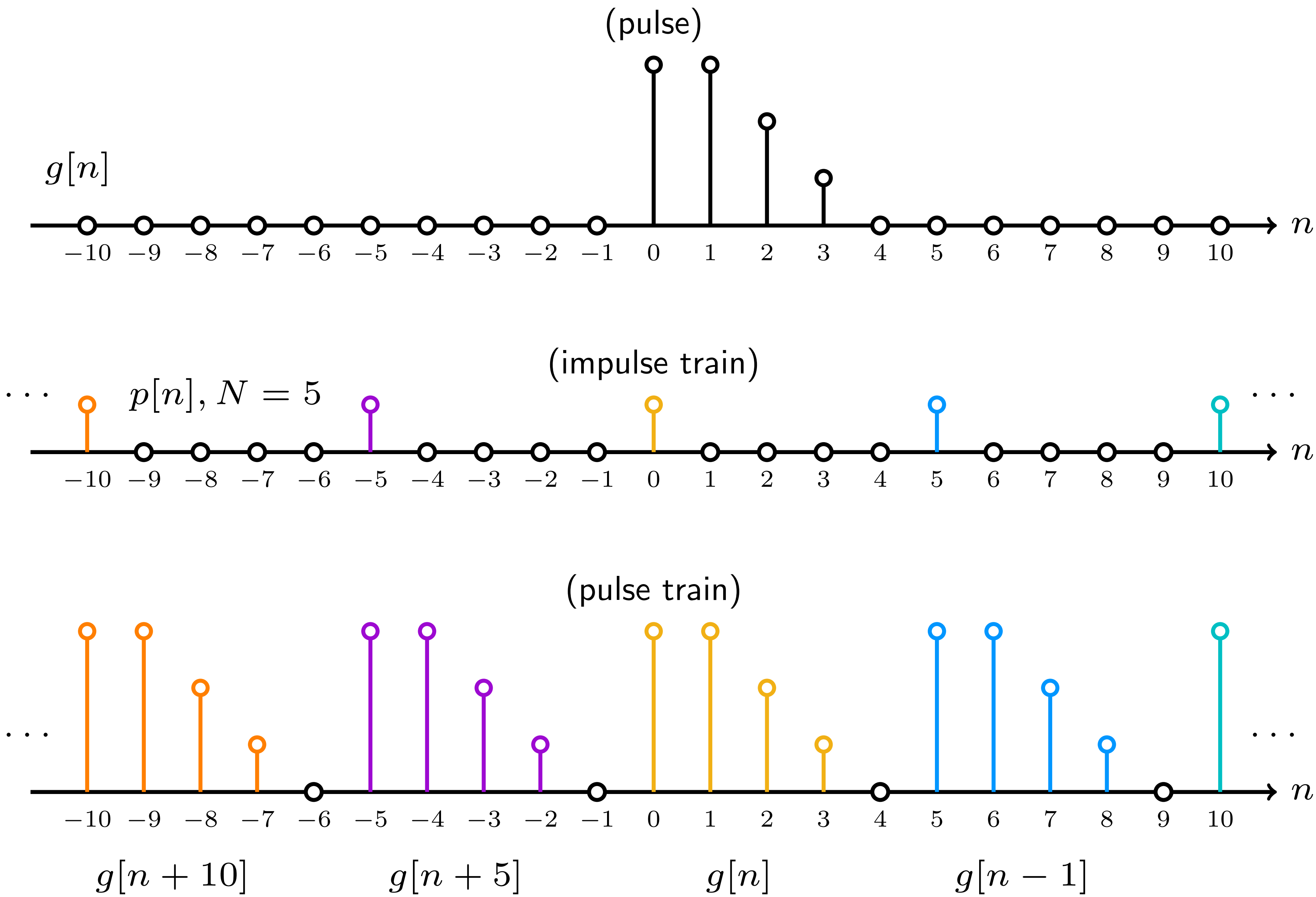
Periodic impulse train (period N):

$$p[n] = \sum_k \delta[n - kN]$$

Periodic pulse train (period N):

$$x[n] = g[n] * p[n] = \sum_k g[n] * \delta[n - kN] = \sum_k g[n - kN]$$

impulse train and pulse train construction



Dirac and Kronecker delta

sifting property:

$$x(t)\delta(t-\tau) = x(\tau)\delta(t-\tau)$$

$$x[n]\delta[n-k] = x[k]\delta[n-k]$$

convolution property:

$$x(t) * \delta(t-\tau) = x(t-\tau)$$

$$x[n] * \delta[n-k] = x[n-k]$$

periodic impulse trains:

$$p(t) = \sum_k \delta(t-kT) \qquad \text{(periodic with period T)}$$

$$p[n] = \sum_k \delta[n-kN] \qquad \text{(periodic with period N)}$$

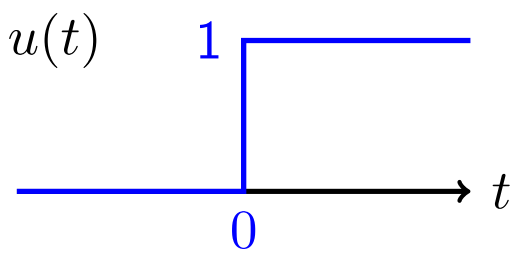
periodic pulse trains:

$$x(t) = g(t) * p(t) = \sum_k g(t - kT) \qquad \text{(periodic with period T)}$$

$$x[n] = g[n] * p[n] = \sum_k g[n - kN] \qquad \text{(periodic with period N)}$$

step functions and properties

step function

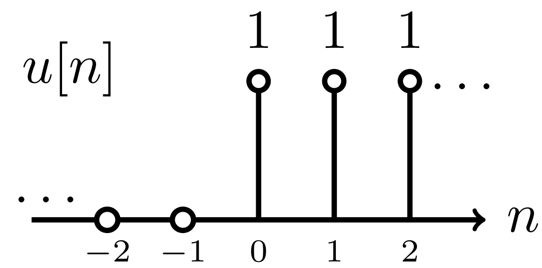


$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

$$u(t) = \int_{-\infty}^{t} \delta(\tau) d\tau$$

$$\delta(t) = \frac{d}{dt} u(t)$$

step function



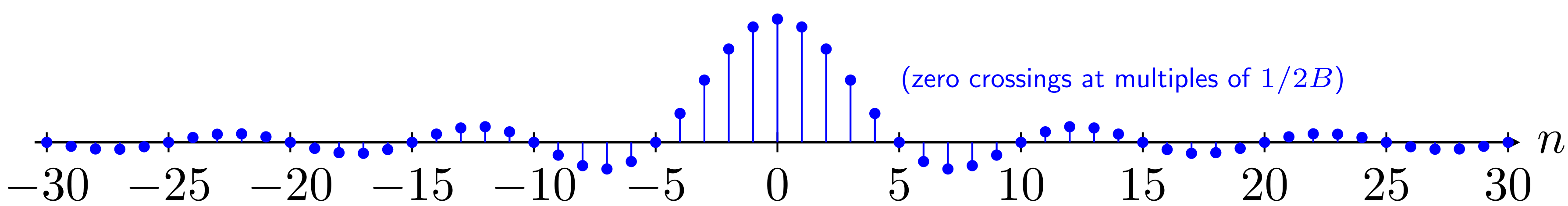
$$u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$$

$$u[n] = \sum_{k=-\infty}^{n} \delta[k]$$

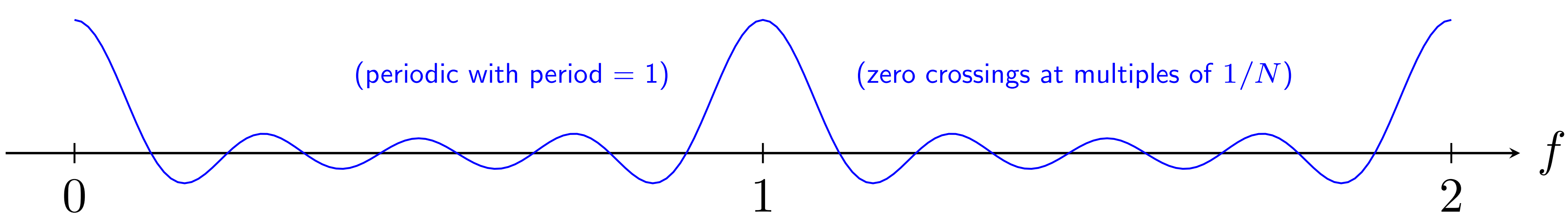
$$\delta[k] = u[n] - u[n-1]$$

sinc

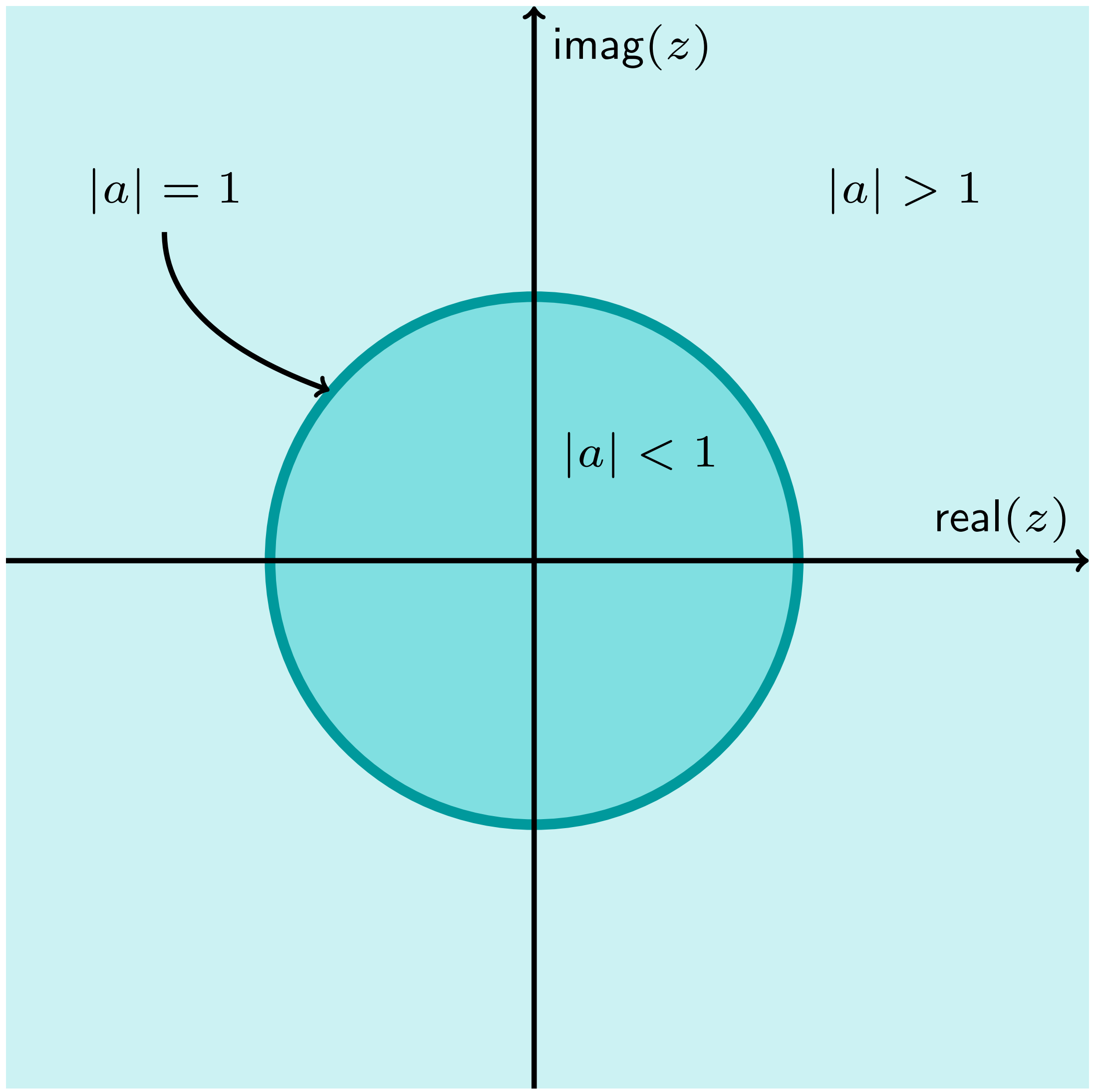
(aperiodic sinc)
$$\begin{cases} \dfrac{\sin(2B\pi n)}{\pi n}, & n \neq 0 \\ 2B, & n = 0 \end{cases} \qquad B = \frac{1}{10}$$



(periodic sinc)
$$\begin{cases} \dfrac{\sin(\pi f N)}{\sin(\pi f)}, & f \neq 0, \pm 1, \pm 2, \cdots \\ N, & f = 0, \pm 1, \pm 2, \cdots \end{cases} \qquad N = 9$$



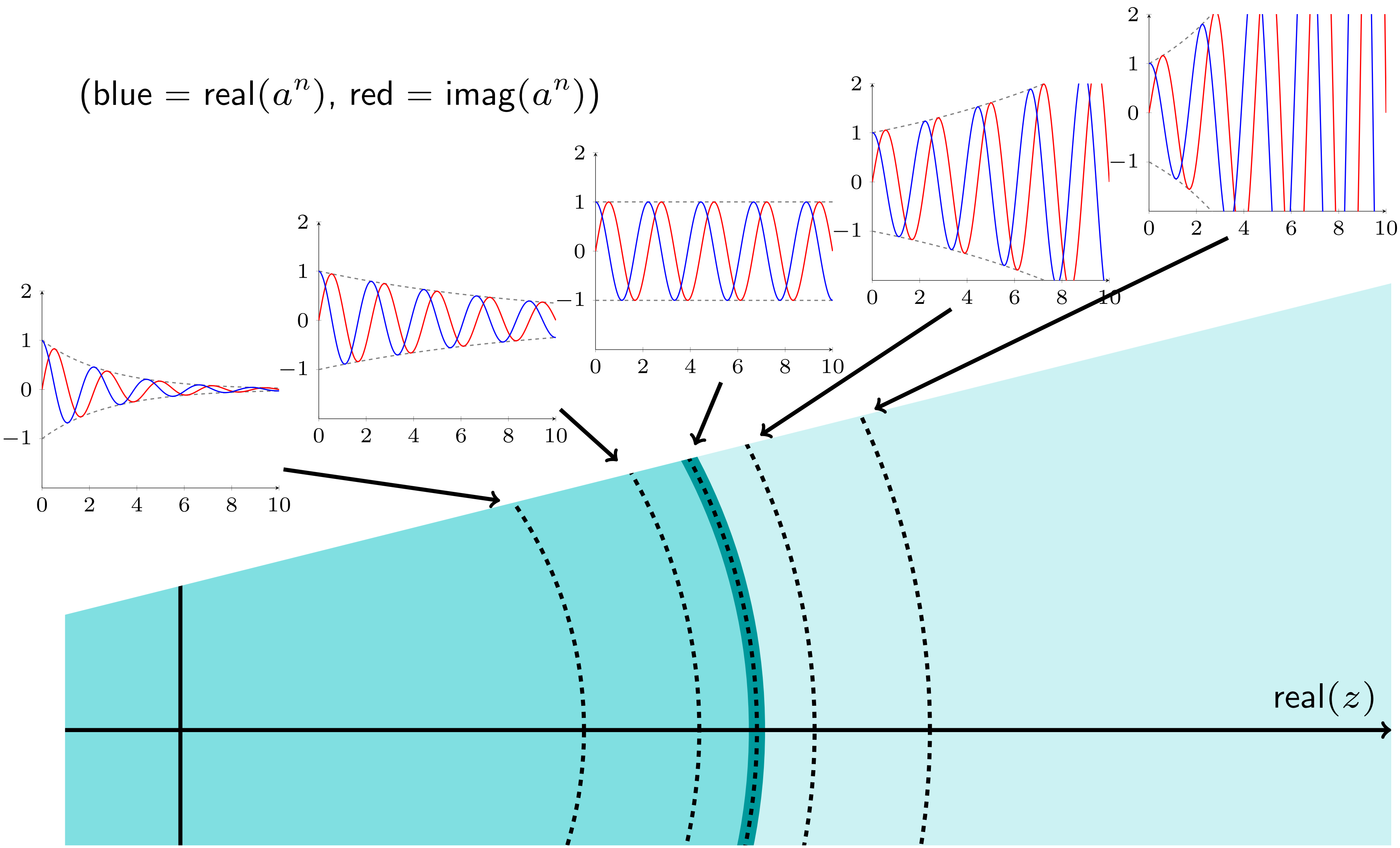
causal exponential $x[n] = a^n u[n]$



(three regions in the complex plane)

envelope of exponential $x[n] = a^n u[n]$

(blue = real(a^n), red = imag(a^n))

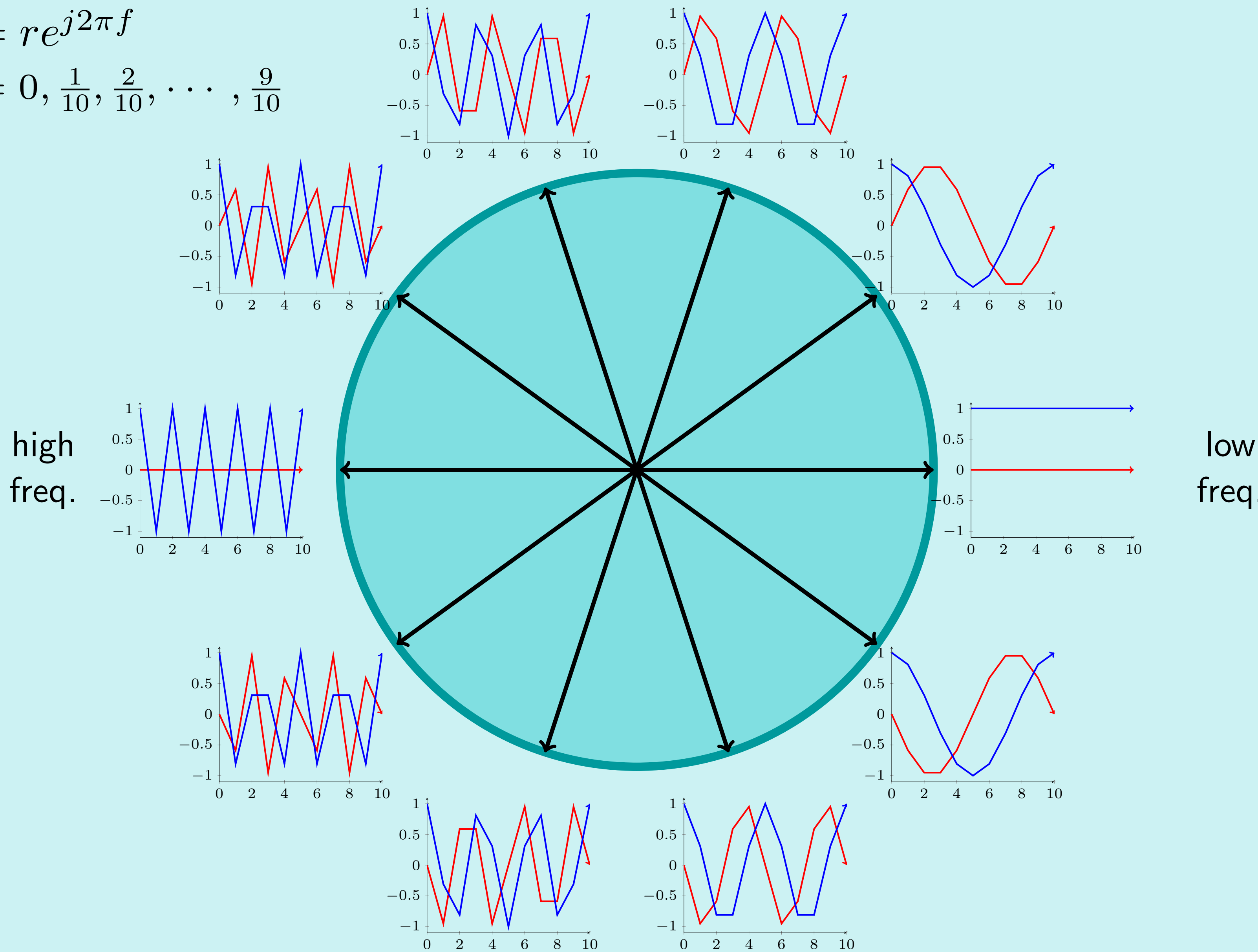


envelope (dashed gray lines) of a^n depends on radius $|a|$

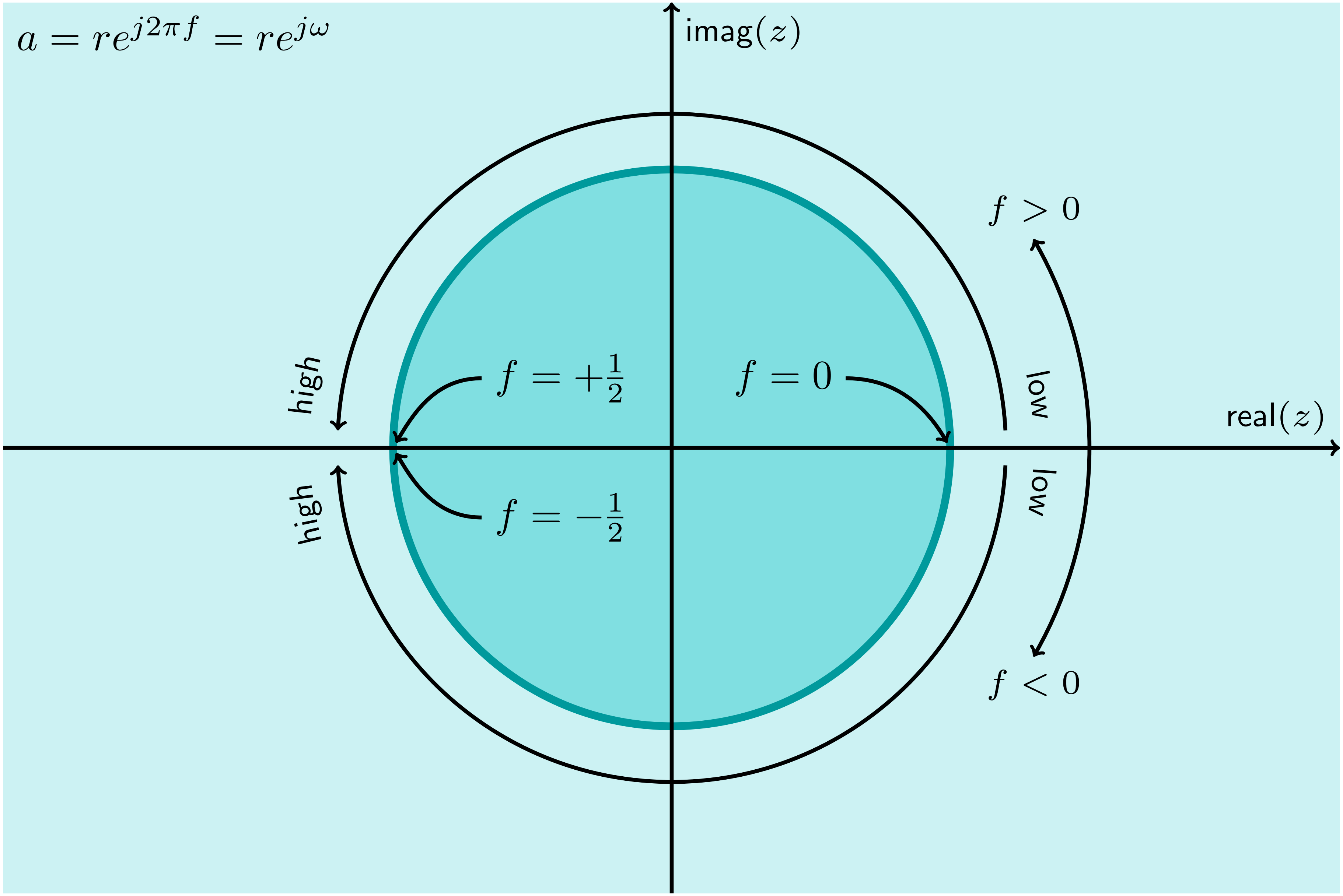
frequency of exponential $x[n] = a^n u[n]$

$a = re^{j2\pi f}$

$f = 0, \frac{1}{10}, \frac{2}{10}, \cdots, \frac{9}{10}$



frequency of exponential $x[n] = a^n u[n]$



polynomials and rational functions

polynomial: (order M, $M+1$ coefficients)

$$H(z) = \sum_{n=0}^{M} h_n z^{-n} = h_0 + h_1 z^{-1} + h_2 z^{-2} + \cdots + h_M z^{-M} = h_0 \prod_{k=1}^{M} (1 - z_k z^{-1})$$

z_k are the zeros of $H(z)$

rational function:

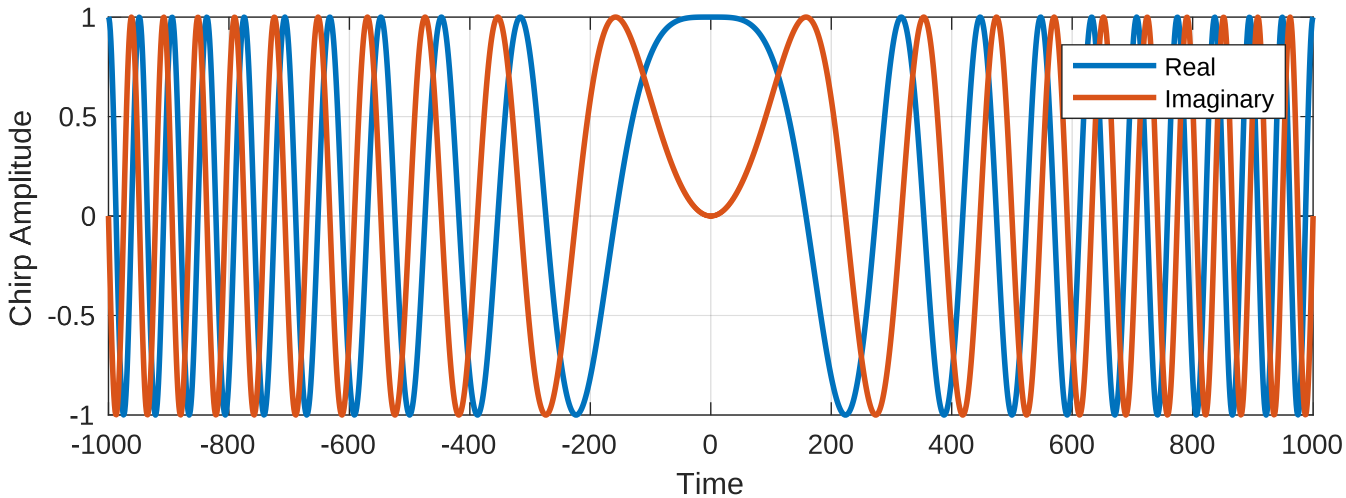
$$H(z) = \frac{\sum_{k=0}^{m} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}} = \frac{b_0}{a_0} \frac{\prod_{k=1}^{M} (1 - z_k z^{-1})}{\prod_{k=1}^{N} (1 - p_k z^{-1})}$$

z_k are the zeros of $H(z)$
p_k are the poles of $H(z)$

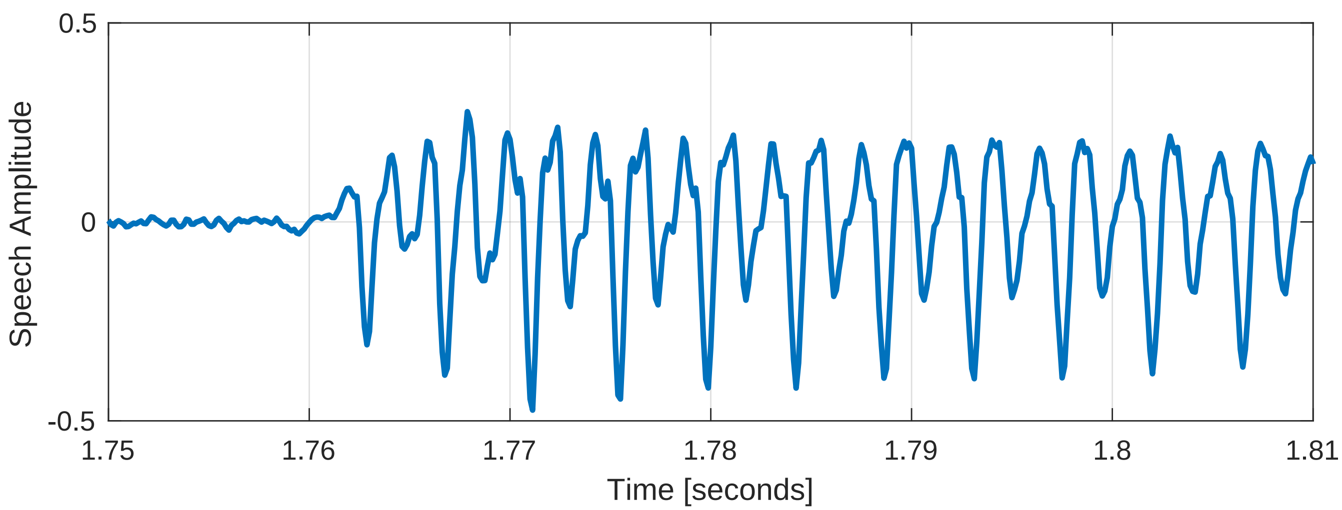
signal classification

deterministic vs. stochastic (random)

- deterministic signal = determined by mathematical formulas, tables, or other rules acting on the independent variable(s)
- stochastic signal = governed random processes that are not completely predictable

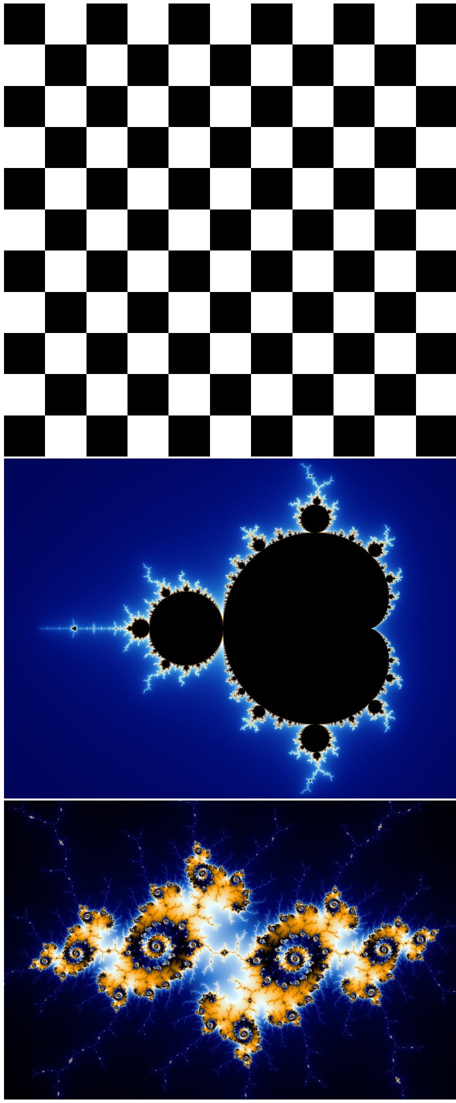


$$\exp\left(j2\pi 10^{-5}n^2\right)$$
$$-1000 \leq n \leq 1000$$
(deterministic)



speech
(random)

deterministic vs. stochastic (random)



(deterministic)

(stochastic)

deterministic vs. stochastic (random)

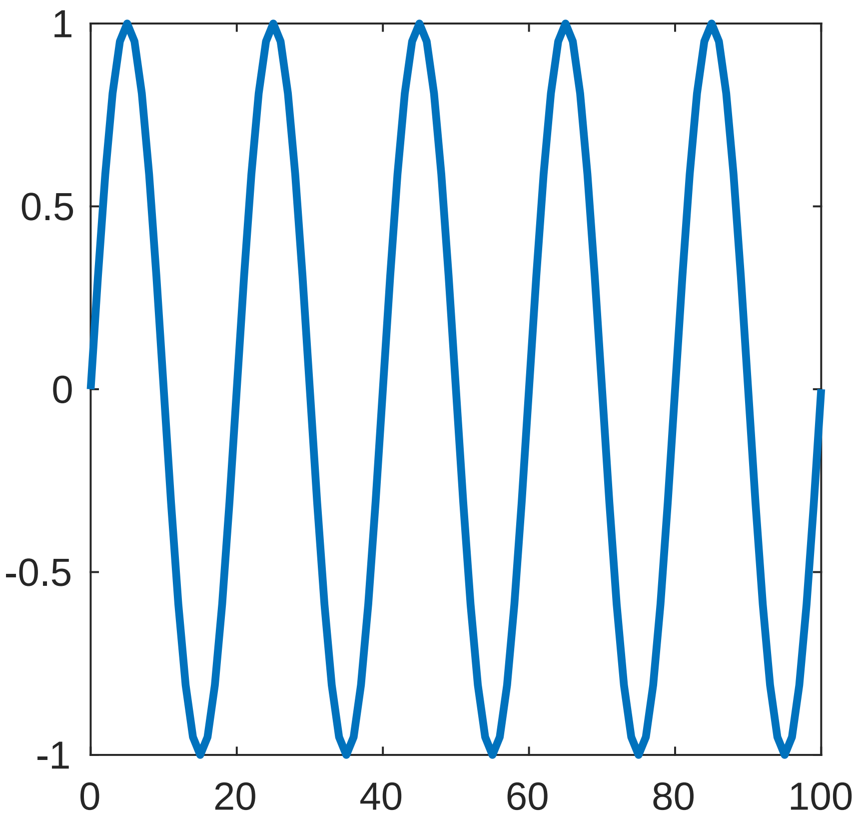
Can calculate by hand (transforms, convolution) with some deterministic signals.

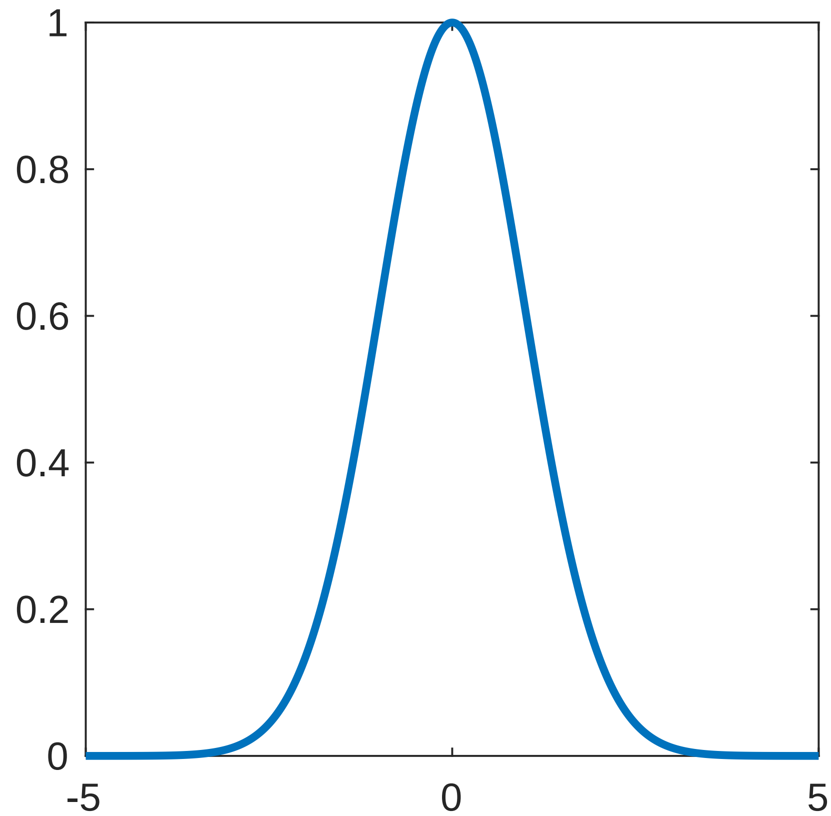
Use computers to calculate with random signals.

We will do both hand calculations and algorithm implementation in software.

periodic vs. aperiodic

- periodic: $x(t + kT) = x(t)$ for all $t \in \mathbb{R}, k \in \mathbb{Z}$
- aperiodic: no repeating patterns





periodic vs. aperiodic

Periodic signals can be constructed from aperiodic signals

Suppose $p(t)$ is aperiodic and $\int p^2(t)dt < \infty$, then

$$x(t) = \sum_{k=-\infty}^{\infty} p(t - kT),$$

is periodic with periodi T.

Suppose $p[n]$ is aperiodic, then

$$x[n] = p[n \text{ mod } N]$$

is periodic with period N, where

$$n \text{ mod } N = n - N\lfloor n/N \rfloor = m, \quad n = lN + m, \;\; m = 0, 1, 2, \cdots, N-1.$$

finite vs. infinite

The “duration” or “width” of a signal is the size of the support set of the independent variable(s).

doubly infinite signal $f(t)$ for $-\infty < t < \infty$

semi-infinte (one-sided) signal $f(t)$ for $0 \leq t < \infty$

semi-infinite (one-sided) signal $f[n]$ for $-\infty < n \leq n_0$

finite signal $f[m, n]$ for $0 \leq m \leq M - 1$ and $0 \leq n \leq N - 1$

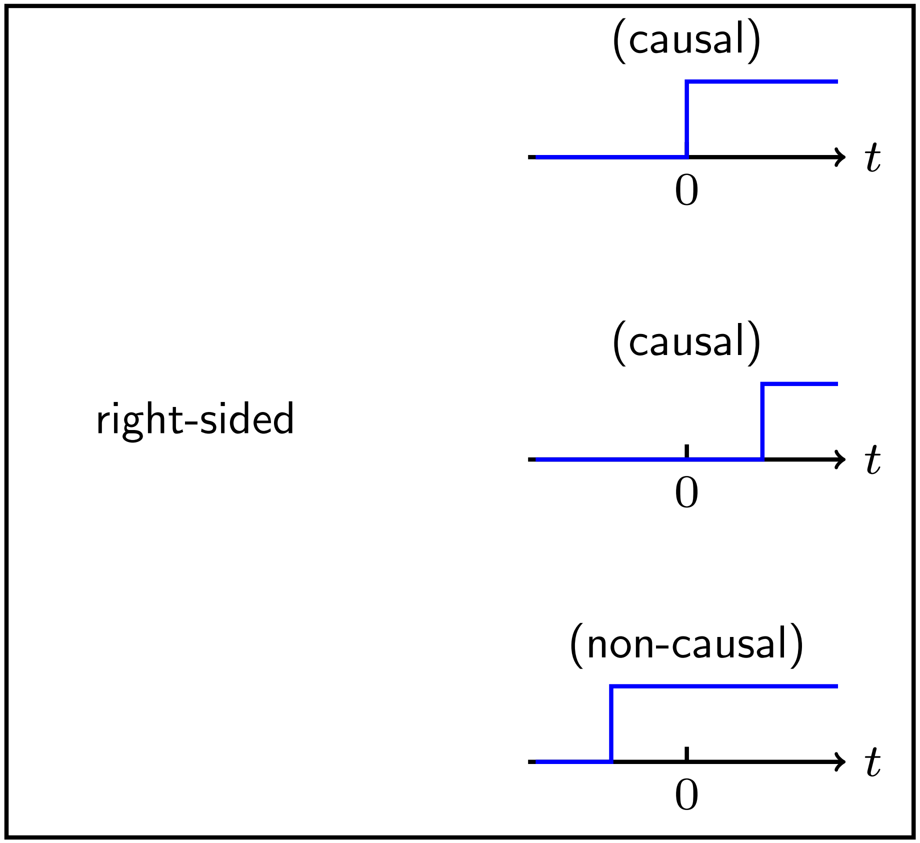
finite 2D signal $=$ an image

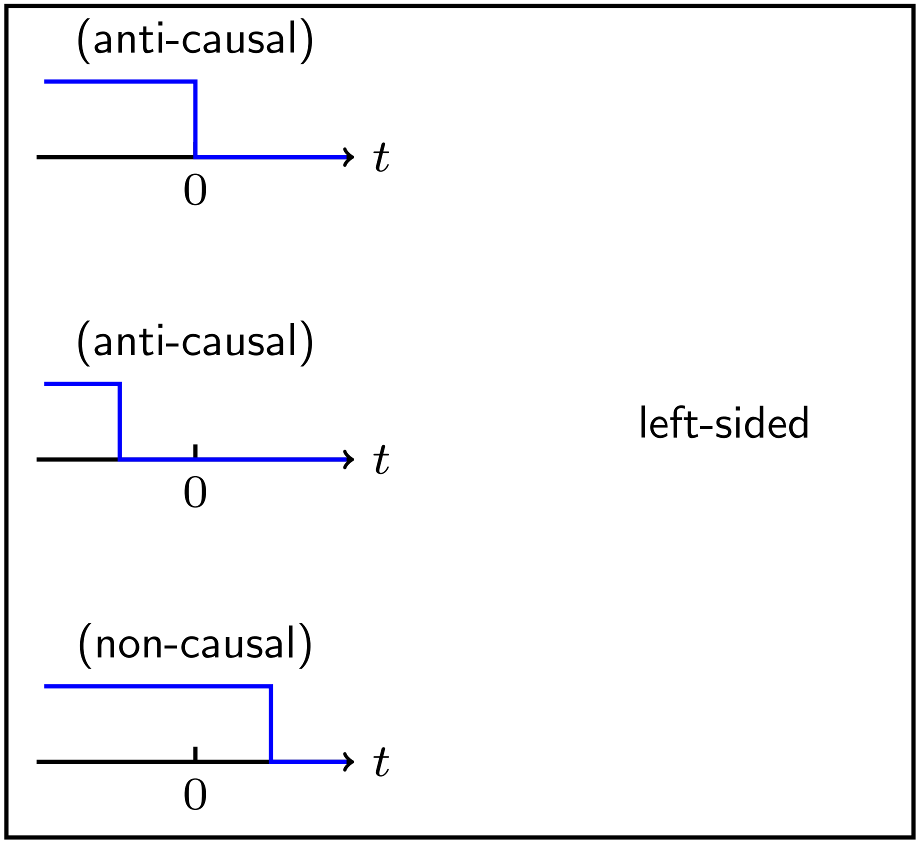
finite 1D signal $=$ your favorite song

semi-infinite 1D signal $=$ your favorite song stuck on repeat

causal vs. non-causal

- Causality describes the support set of a signal.
- Causality usually describes signals where the independent variable is related to time.
- Causal: $f(t) = 0$ for $t < 0$
- Anti-causal: $f(t) = 0$ for $t > 0$

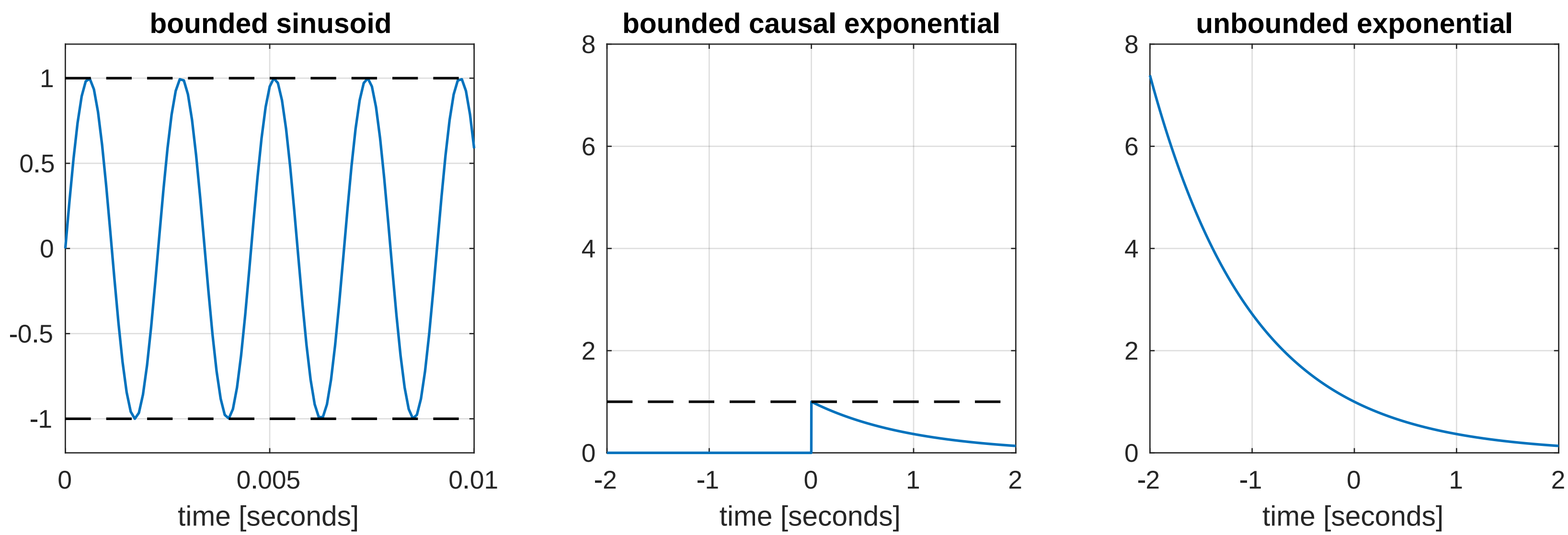




bounded vs. unbounded

If there exists $M < \infty$ such that $|x(t)| \leq M$ for all t, then $x(t)$ is bounded. Otherwise it is unbounded.

- bounded: $x(t) = \sin(2\pi 440 t)$, $M = 1$
- bounded: $x(t) = e^{-t}u(t)$, $M = 1$
- unbounded: $x(t) = e^{-t}$ (blows up as $t \to -\infty$)
- unbounded: $x[n] = x[n-1] + x[n-2], n \geq 0, x[0] = 1, x[1] = 1$ (Fibonacci sequence)



symmetry

symmetry name	mathematical definition	example
even	$x(-t) = x(t)$	$\frac{\sin(\pi B t)}{\pi t}$
odd	$x(-t) = -x(t)$	$\sin(2\pi F_0 t)$
real	$x^*(t) = x(t)$	$u(t)$
imaginary	$x^*(t) = -x(t)$	$x(t) = \frac{j}{\pi t}$
Hermitian	$x^*(-t) = x(t)$	$X(F) = \frac{e^{j2\pi F T} - 1}{j2\pi F}$
anti-Hermitian	$x^*(-t) = -x(t)$	$X(F) = \begin{cases} -1 + j, & F > 0 \\ 0, & F = 0, \\ +1 + j, & F < 0 \end{cases}$

summability

absolute sum of $x[n]$

$$A_x = \sum_{n=-\infty}^{\infty} |x[n]|$$

If $A_x < \infty$, then $x[n]$ is absolutely summable

energy of $x[n]$

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

If $E_x < \infty$, then $x[n]$ is an energy signal

power of $x[n]$

$$P_x = \lim_{N\to\infty} \frac{\sum_{n=-N}^{N} |x[n]|^2}{2N+1}$$

If $P_x < \infty$, then $x[n]$ is a power signal

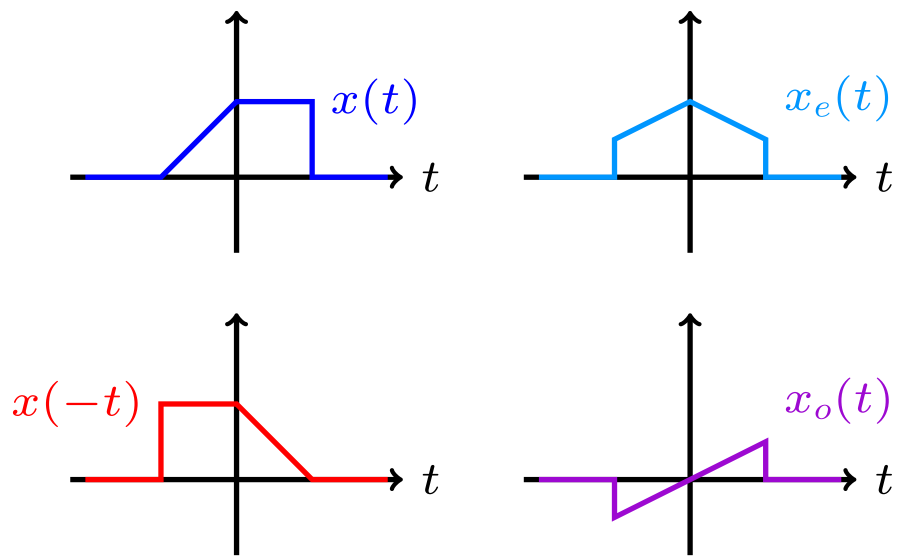
signal classification summary

attribute	**class**
predictability	deterministic, random
repeating structure	periodic, aperiodic
support set	finite, semi-infinite, infinite
amplitude limits	bounded, unbounded
causality	causal, anticausal, non-causal, left-sided, right-sided
symmetry	even, odd, Hermitian, anti-Hermitian
summability	absolutely summable, energy signal, power signal

signal decompositions

even & odd

$$x(t) = x_e(t) + x_o(t), \qquad \begin{aligned} x_e(t) &= \frac{1}{2}\left[x(t) + x(-t)\right] \\ x_o(t) &= \frac{1}{2}\left[x(t) - x(-t)\right] \end{aligned}$$



real & imaginary

$$x(t) = x_r(t) + jx_i(t), \qquad \begin{aligned} x_r(t) &= \frac{1}{2}\left[x(t) + x^*(t)\right] \\ x_i(t) &= \frac{1}{2j}\left[x(t) - x^*(t)\right] \end{aligned}$$

Hermitian & anti-Hermitian

$$x(t) = x_h(t) + jx_a(t), \qquad \begin{aligned} x_h(t) &= \frac{1}{2}\left[x(t) + x^*(-t)\right] \\ x_a(t) &= \frac{1}{2j}\left[x(t) - x^*(-t)\right] \end{aligned}$$

four-way decomposition

$$
\begin{aligned}
x(t) &= x_{re}(t) + x_{ro}(t) + jx_{ie}(t) + jx_{io}(t) \\
x_{re}(t) &= \frac{1}{4}\left[x(t) + x(-t) + x^*(t) + x^*(-t)\right] \\
x_{ro}(t) &= \frac{1}{4}\left[x(t) - x(-t) + x^*(t) - x^*(-t)\right] \\
x_{ie}(t) &= \frac{1}{4j}\left[x(t) + x(-t) - x^*(t) - x^*(-t)\right] \\
x_{io}(t) &= \frac{1}{4j}\left[x(t) - x(-t) - x^*(t) + x^*(-t)\right]
\end{aligned}
$$

assignment

1. What are the dimensions and number of channels of the following signals:

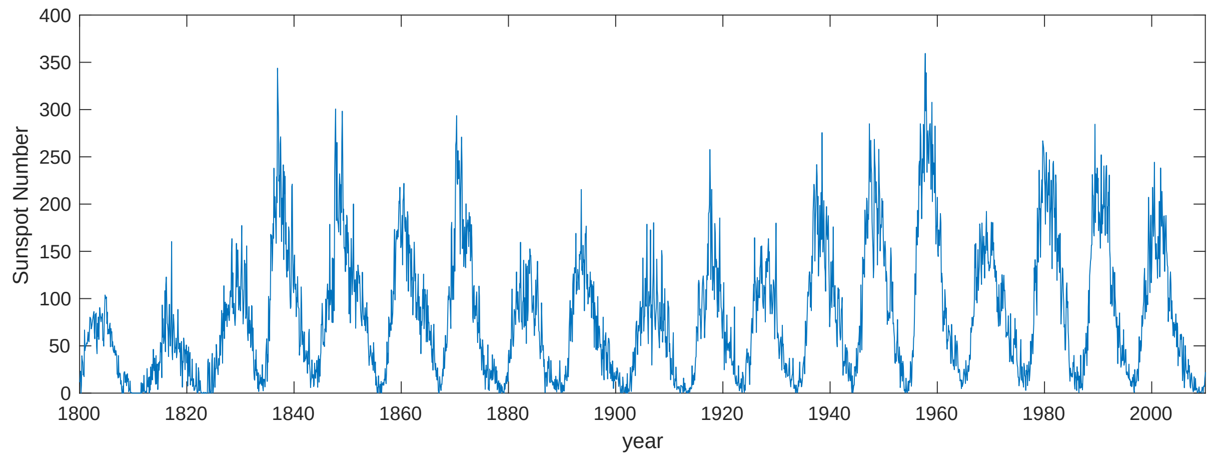
(a) Color video

$$\begin{bmatrix} v_{\text{red}}(x, y, t) \\ v_{\text{green}}(x, y, t) \\ v_{\text{blue}}(x, y, t) \end{bmatrix}$$

(b) ITU 5.1 surround sound

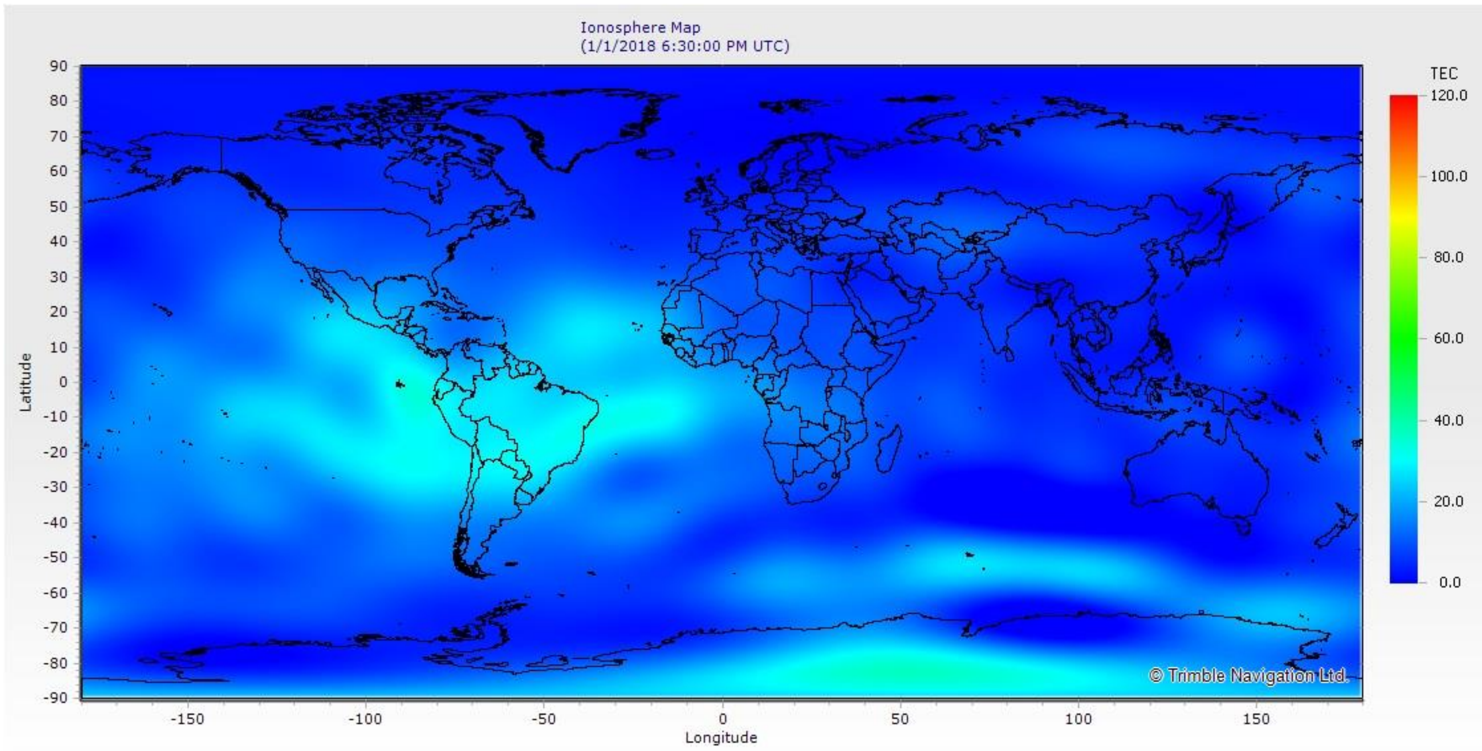
$$\begin{bmatrix} x_{\text{center}}(t) \\ x_{\text{left}}(t) \\ x_{\text{right}}(t) \\ x_{\text{left surround}}(t) \\ x_{\text{right surround}}(t) \\ x_{\text{subwoofer}}(t) \end{bmatrix}$$

2. The plot below shows the number of sunspots observed per year.



Describe the independent (domain) and dependent (range) variables of this signal using the terms "continuous" and "discrete".

3. The plot below shows the total electron concentration in the Ionosphere at some point in time.



Describe the independent (domain) and dependent (range) variables of this signal using the terms “continuous” and “discrete”.

4. Given $x[n] = 0.9^n u[n]$, sketch the following signals:

   (a) $x[n-10]$
   (b) $x[n+10]$
   (c) $x[-n]$
   (d) $x[-n+10]$
   (e) $x[-n-10]$

5. Use the sifting property to simplify: $\cos(2\pi 0.3n)\delta[k+20] =?$

6. Use the sifting property to simplify the convolution sum: $x[n] * \delta[n] = \sum_k x[k]\delta[n-k] =?$

7. Use the sifting property to simplify the convolution sum: $x[n] * \delta[n-m] = \sum_k x[k]\delta[n-m-k] =?$

8. Let $g[n] = (n+1)\,(u[n] - u[n-4])$ and

$$p[n] = \sum_{k=-\infty}^{\infty} \delta[n-6k]$$

   Sketch a picture of $g[n] * p[n]$ for $-20 \le n \le 20$.

9. Let $a = 0.9e^{j2\pi 0.1}$.
   (a) Plot the point a on the complex plane. Include the unit circle.
   (b) Sketch $x[n] = a^n u[n]$ for $-10 \leq n \leq 10$.

10. Let $a = 0.8899 + j0.6466$.
    (a) Plot the point a on the complex plane. Include the unit circle.
    (b) Sketch $x[n] = a^n u[n]$ for $-10 \leq n \leq 10$.

11. Prove that $x[n] = a^n u[n]$ is absolutely summable when $|a| < 1$.

12. Let $a = 0.9e^{j2\pi 0.1}$. Compute the energy in $x[n] = a^n u[n]$.

13. $x[n] = \sin(\pi f n)/(\pi n)$ is not absolutely summable. Prove that it is an energy signal.

14. Compute the power in $x[n] = \cos(2\pi \frac{7}{20} n)$.

15. Compute the energy and power of $u[n]$.

16. Use Matlab's `roots` function to calculate the zeros of the polynomial $H(z) = 1 - z^{-1} - z^{-2}$. (Hint: Enter `help roots` at the Matlab command prompt.)

17. Let $x[n] = e^{j2\pi f n}$. Use the definitions on the signal decomposition slides to compute the following:

    - $x_e[n]$ (even part)
    - $x_o[n]$ (odd part)
    - $x_r[n]$ (real part)
    - $x_i[n]$ (imaginary part)
    - $x_h[n]$ (Hermitian part)
    - $x_a[n]$ (anti-Hermitian part)