ECE 3640 - Discrete-Time Signals and Systems
Acoustic Reverberation

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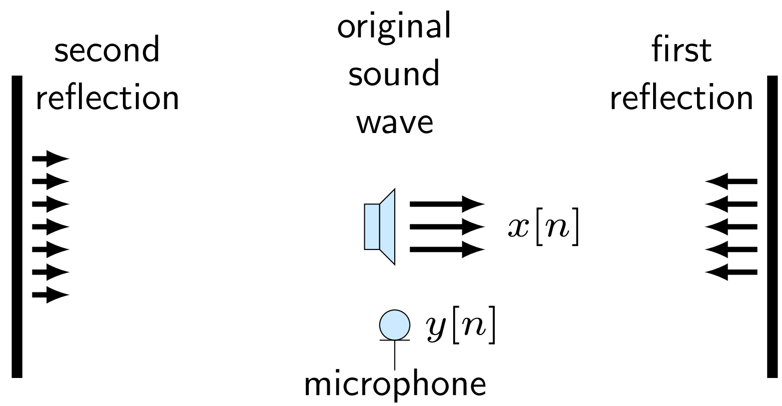
repeated sound wave model



- measured signal:

$$y[n] = x[n] + Ax[n-d] + A^2x[n-2d] + \cdots + A^kx[n-kd] + \cdots$$
$$= \sum_{k=0}^{\infty} A^k x[n-kd]$$

- d samples of delay
- A attenuation, $0 < A < 1$

signal processing model

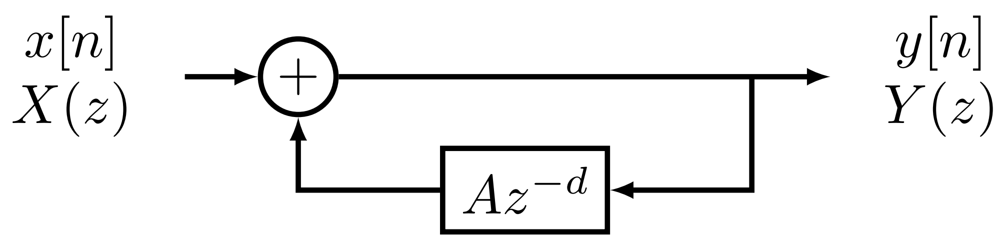
- take z-transforms:

$$Y(z) = \frac{1}{1 - Az^{-d}} X(z)$$

$$Y(z)(1 - Az^{-d}) = X(z) \qquad H(z) = \frac{1}{1 - Az^{-d}}$$

$$\boxed{Y(z) = X(z) + Az^{-d} Y(z)}$$

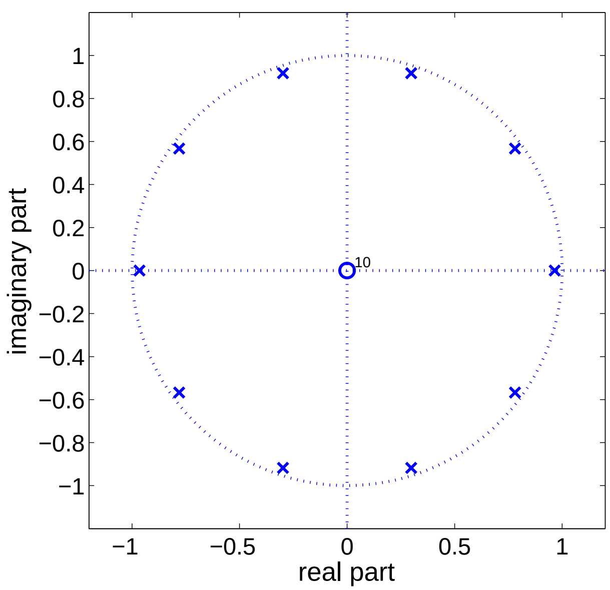
- block diagram:

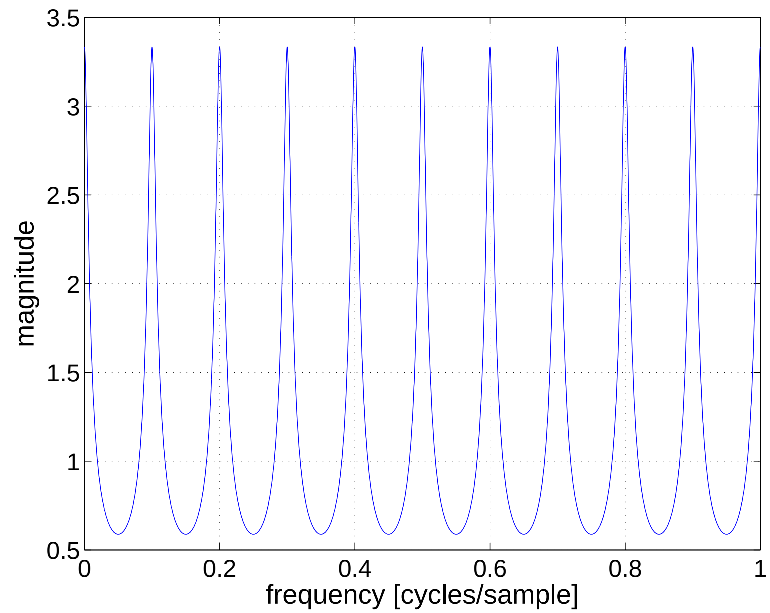


transfer function analysis

$$H(z) = \frac{1}{1 - Az^{-d}} = \frac{z^d}{z^d - A}$$

- zeros: $z_i = 0, \; i = 0, 1, \cdots, d-1$
- poles: $p_i = A^{1/d} e^{j2\pi i/d}, \; i = 0, 1, \cdots, d-1$





example: $A = 0.7, d = 10$

impulse response analysis

- time-domain model:

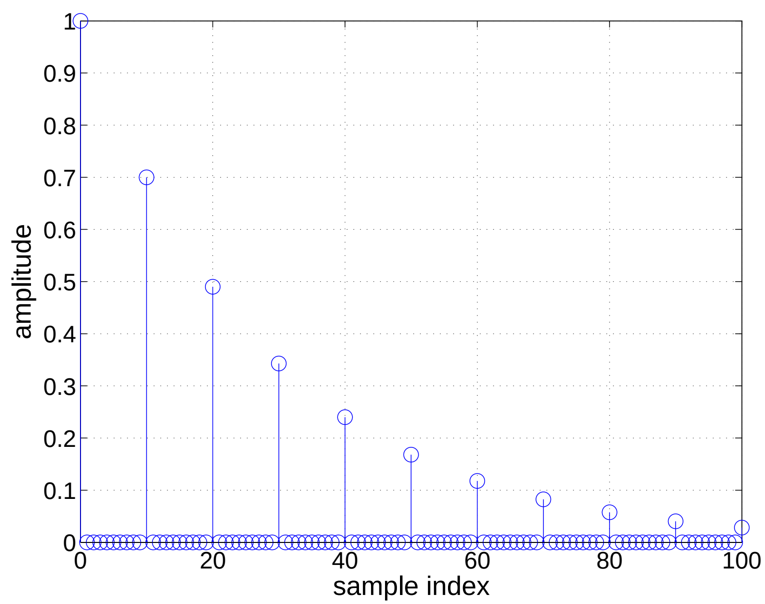
  $y[n] = x[n] + Ax[n-d] + A^2x[n-2d] + \cdots + A^kx[n-kd] + \cdots = \sum_{k=0}^{\infty} A^k x[n-kd]$

- impulse response:

  $h[n] = \delta[n] + A\delta[n-d] + A^2 + \delta[n-2d] + \cdots + A^k\delta[n-kd] + \cdots = \sum_{k=0}^{\infty} A^k \delta[n-kd]$

- example when $d = 4$: $h[n] = \{1, 0, 0, 0, A, 0, 0, 0, A^2, 0, 0, 0, A^3, 0, 0, 0, \cdots\}$

  (there are $d-1$ zeros between powers of A)



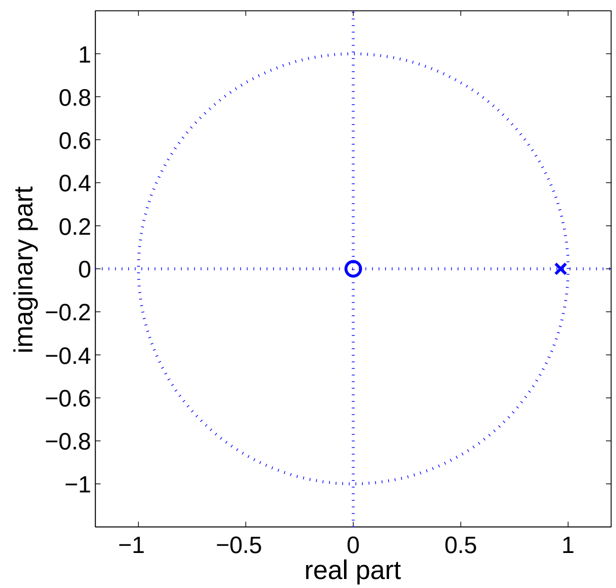
$A = 0.7, \quad d = 10$

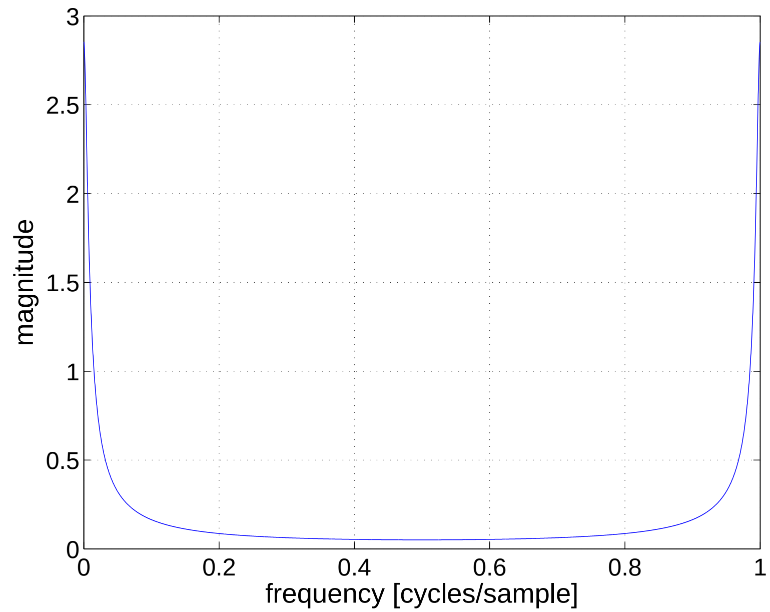
partial fraction expansion

$$
\begin{aligned}
H(z) &= \frac{1}{1-0.7z^{-10}} \\
&= \left(\frac{0.1}{1-0.97z^{-1}}\right) + \left(\frac{0.1}{1+0.97z^{-1}}\right) \\
&+ \left(\frac{0.1(2-1.56z^{-1})}{1-1.56z^{-1}+0.93z^{-2}}\right) \\
&+ \left(\frac{0.1(2-0.60z^{-1})}{1-0.60z^{-1}+0.93z^{-2}}\right) \\
&+ \left(\frac{0.1(2+0.60z^{-1})}{1+0.60z^{-1}+0.93z^{-2}}\right) \\
&+ \left(\frac{0.1(2+1.56z^{-1})}{1+1.56z^{-1}+0.93z^{-2}}\right)
\end{aligned}
$$

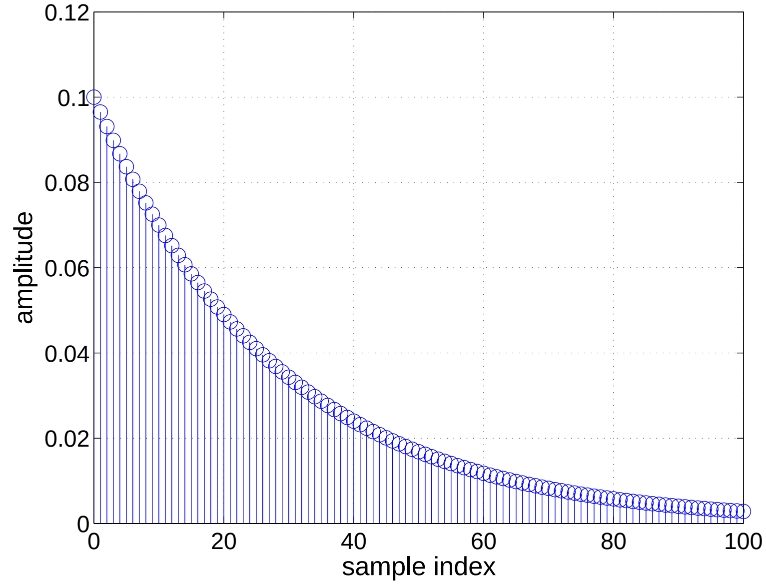
```
A=0.7; d=10; [R,P,K]=residuez(1,[1 zeros(1,d-1) -A]);
% R = residues
% P = poles
% K = direct terms
```

real pole

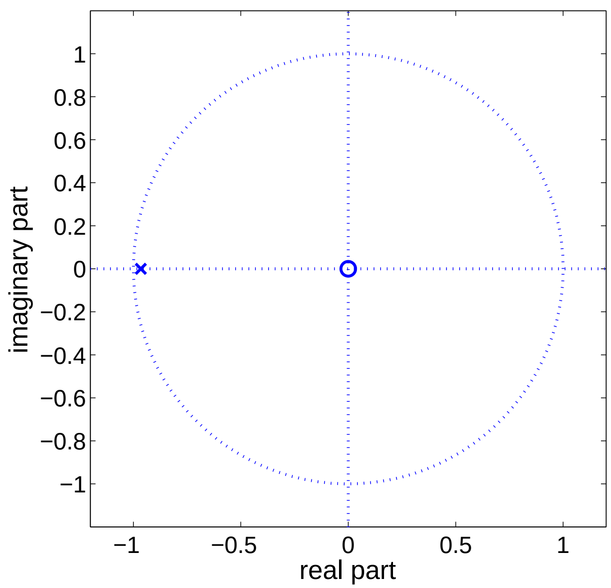


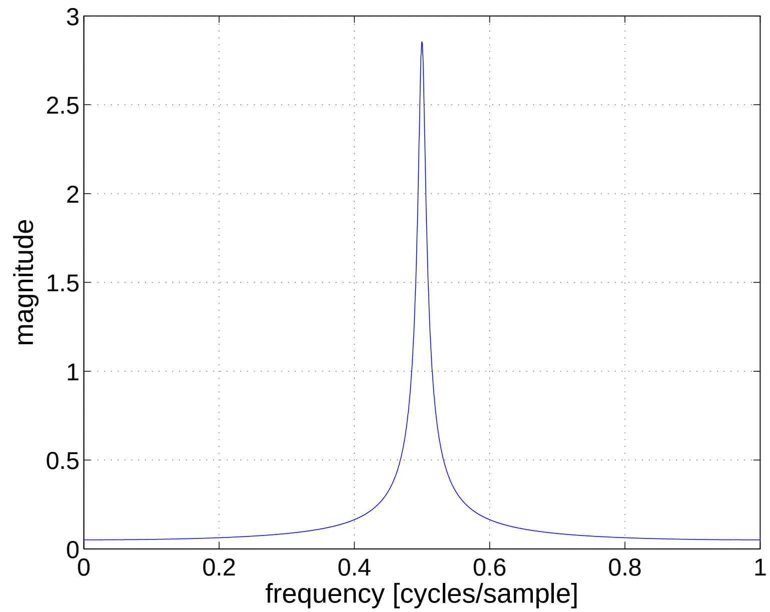


$$\frac{0.1}{1-0.97z^{-1}}$$

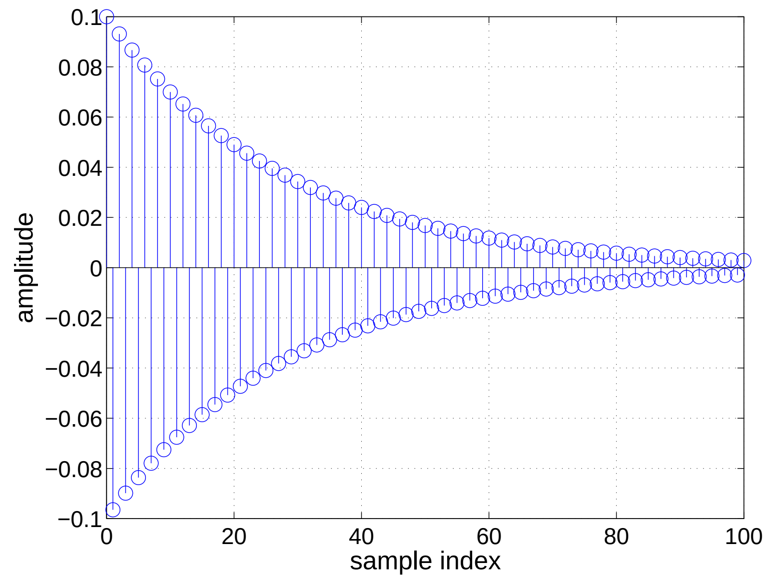


real pole

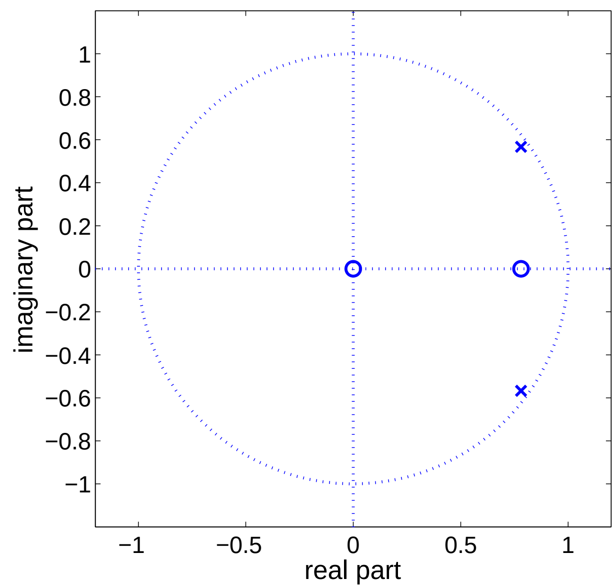


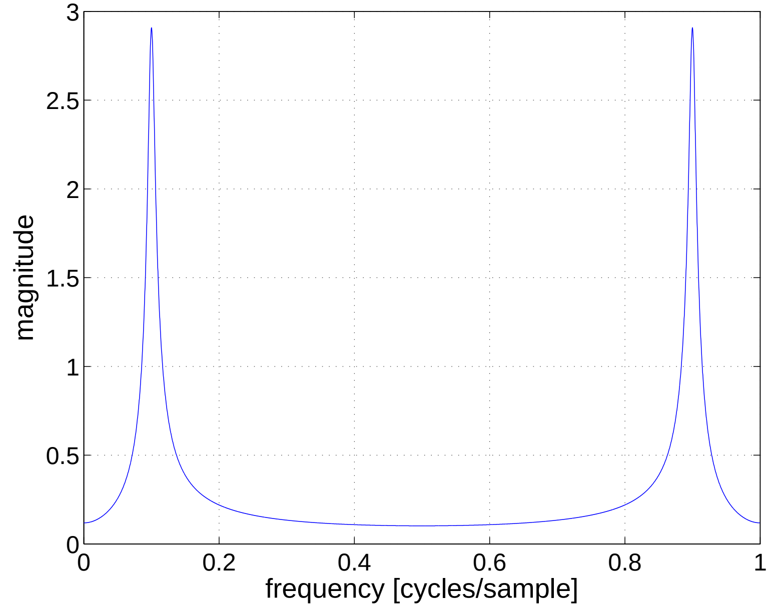


$$\frac{0.1}{1 + 0.97z^{-1}}$$

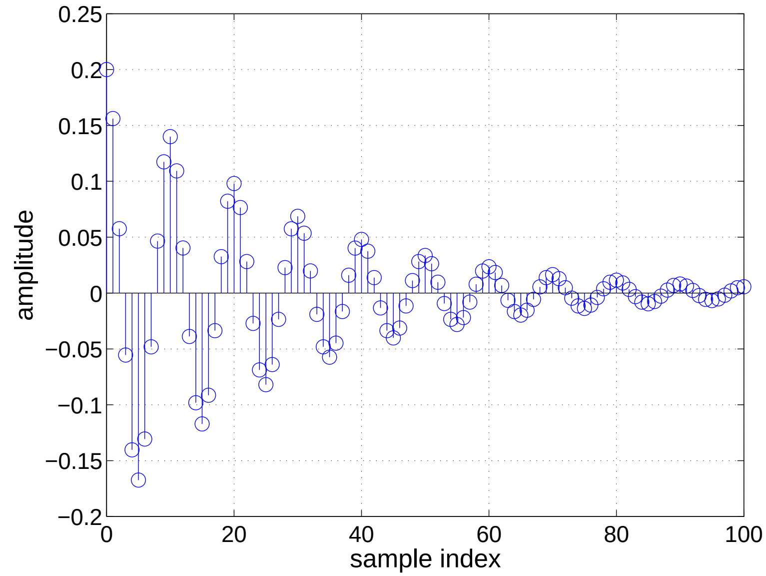


complex conjugate pole pair

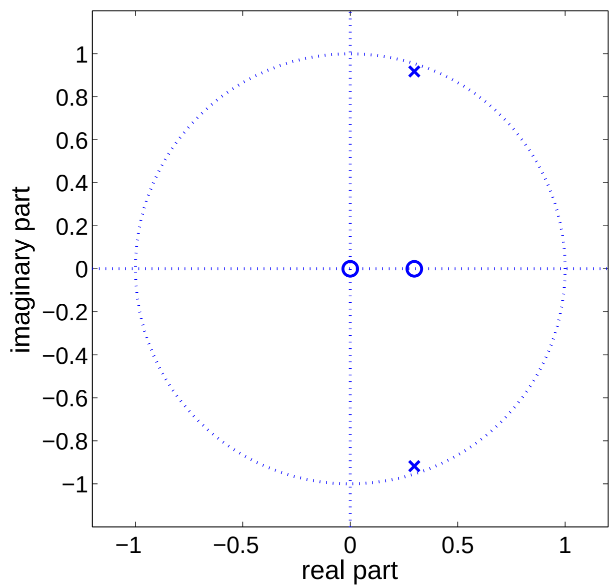


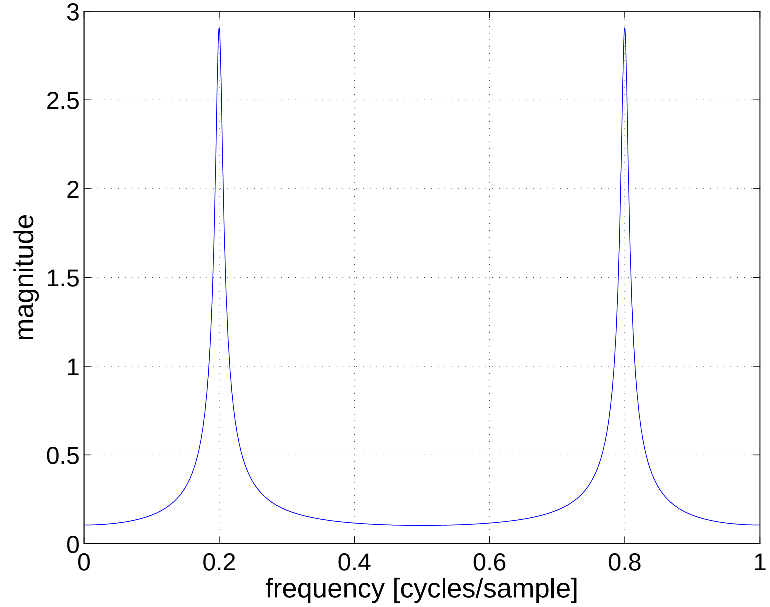


$$\frac{0.1(2 - 1.56z^{-1})}{1 - 1.56z^{-1} + 0.93z^{-2}}$$

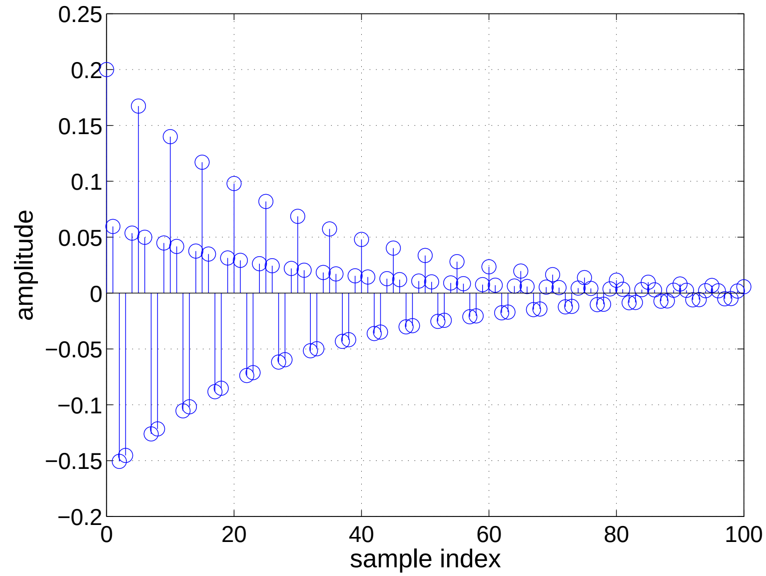


complex conjugate pole pair

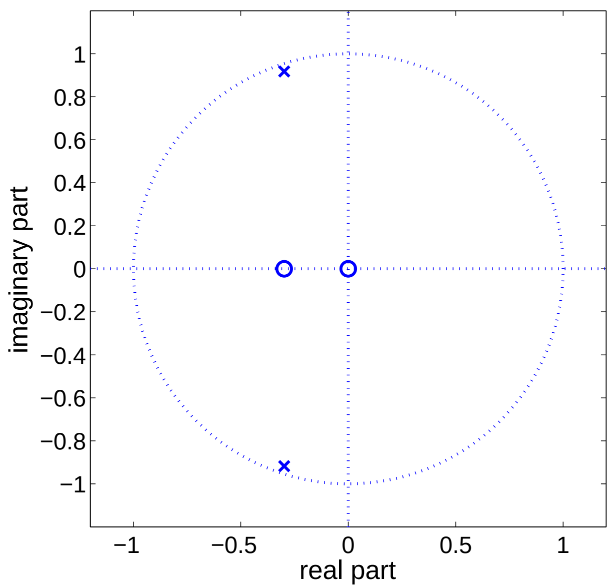


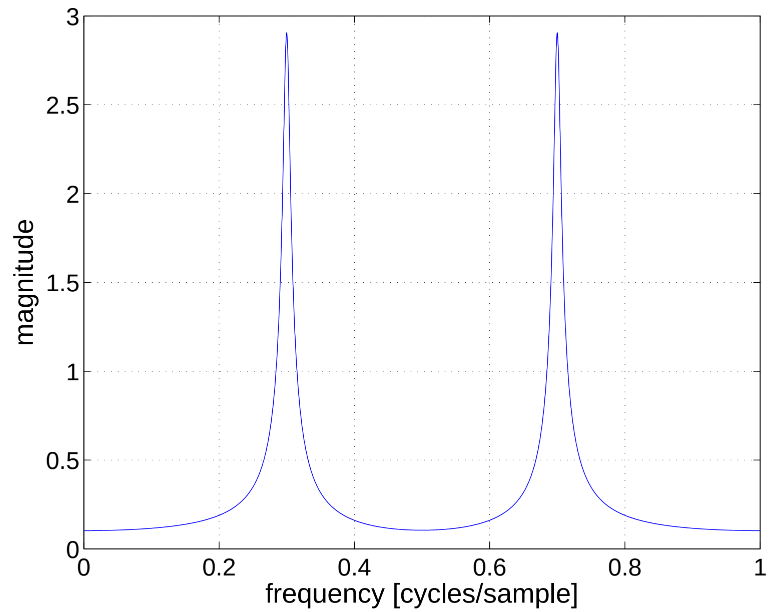


$$\frac{0.1(2 - 0.6z^{-1})}{1 - 0.6z^{-1} + 0.93z^{-2}}$$

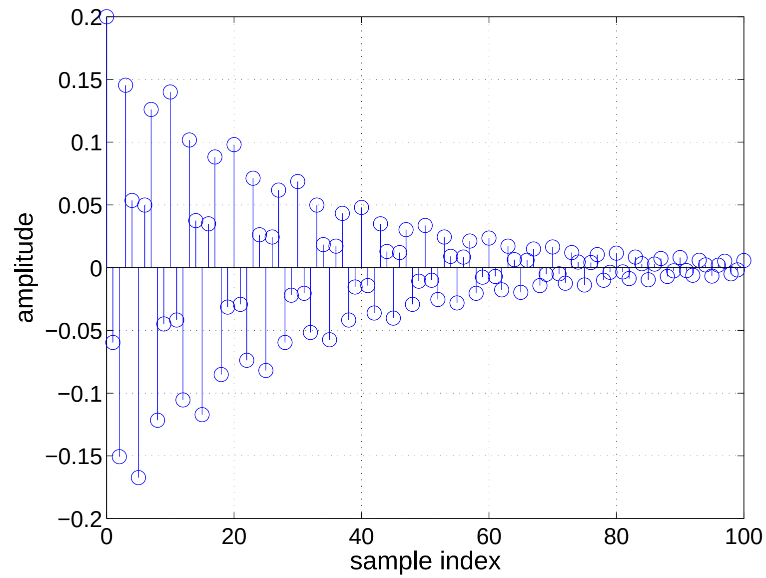


complex conjugate pole pair

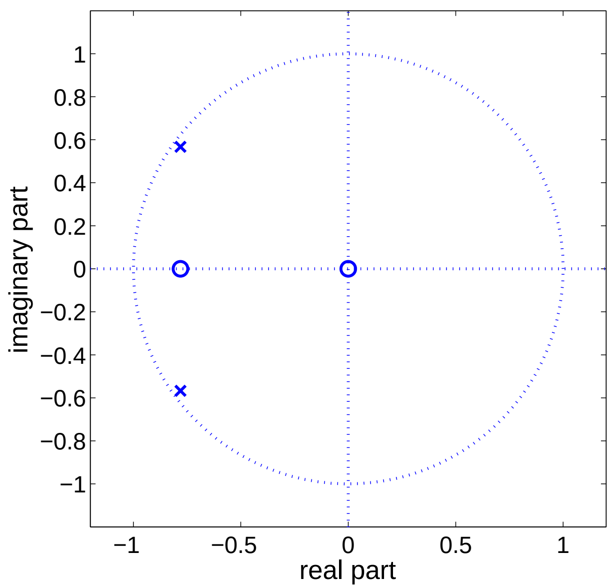


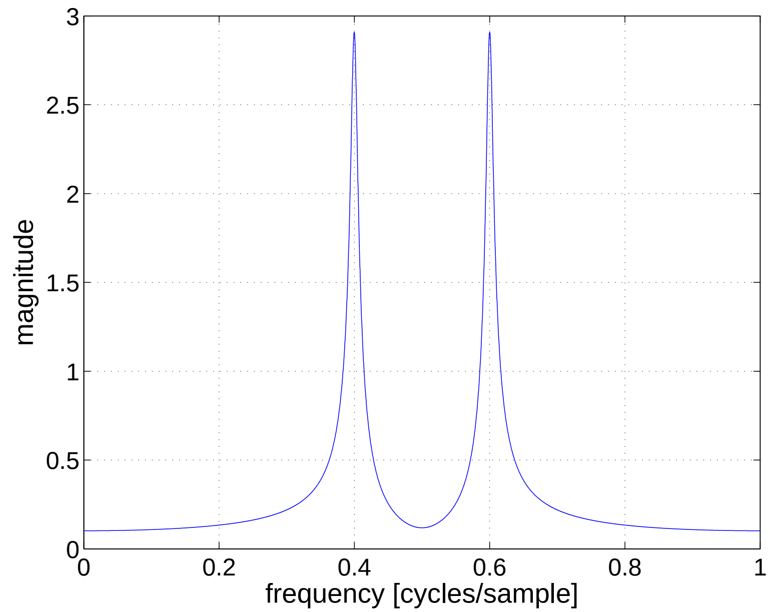


$$\frac{0.1(2 + 0.6z^{-1})}{1 + 0.6z^{-1} + 0.93z^{-2}}$$

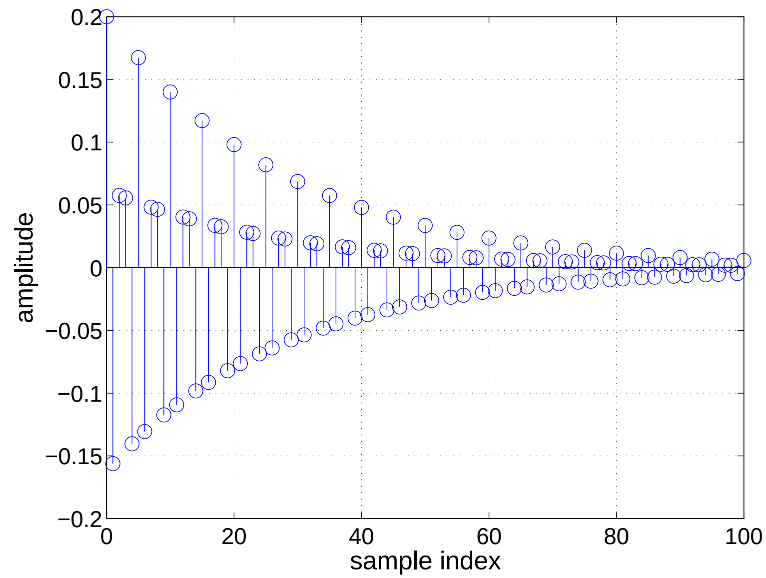


complex conjugate pole pair



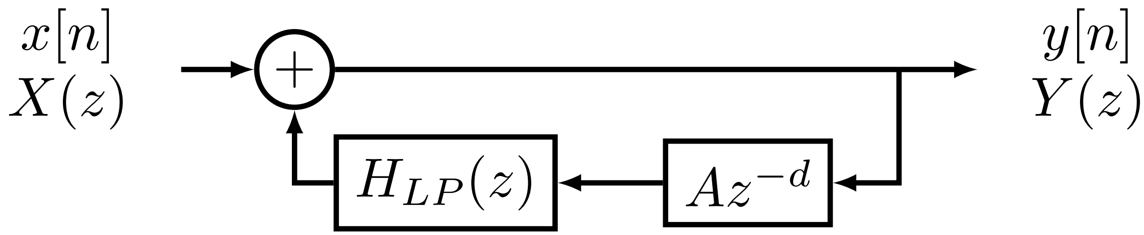


$$\frac{0.1(2 + 1.56z^{-1})}{1 + 1.56z^{-1} + 0.93z^{-2}}$$



include low-pass filter in feedback path

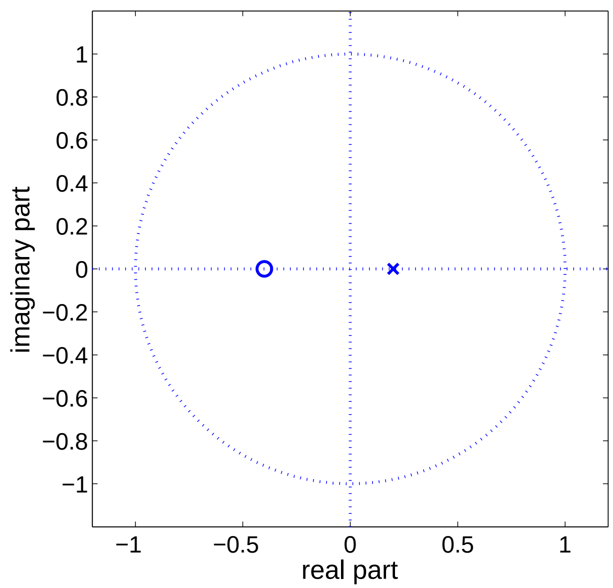
- model the idea that high frequencies are damped out faster than low frequencies
- block diagram:

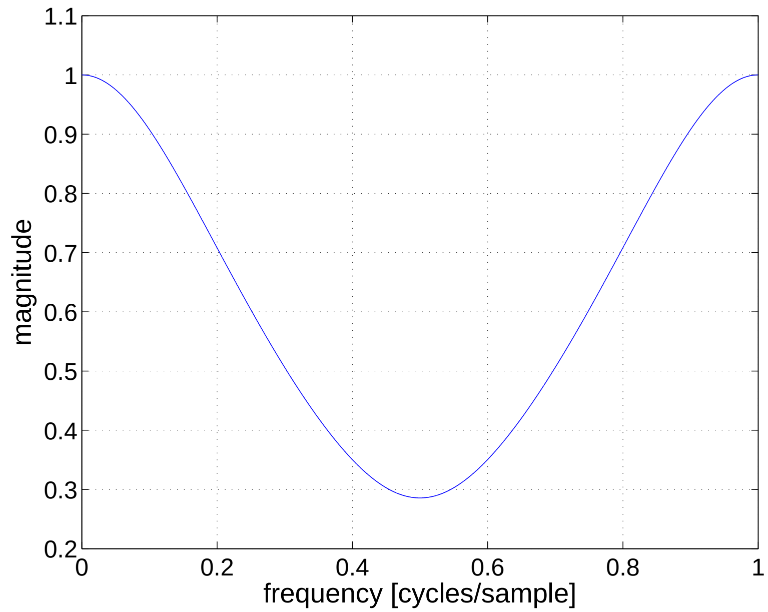


- simple low pass filter: $H_{LP}(z) = \dfrac{K(1 - z_1 z^{-1})}{1 - p_1 z^{-1}}$, $K = \dfrac{1 - p_1}{1 - z_1}$ for unity DC gain
- overall model transfer function

$$H(z) = \frac{1 - p_1 z^{-1}}{1 - p_1 z^{-1} + KAz^{-d} - z_1 KAz^{-(d+1)}}$$

low pass filter

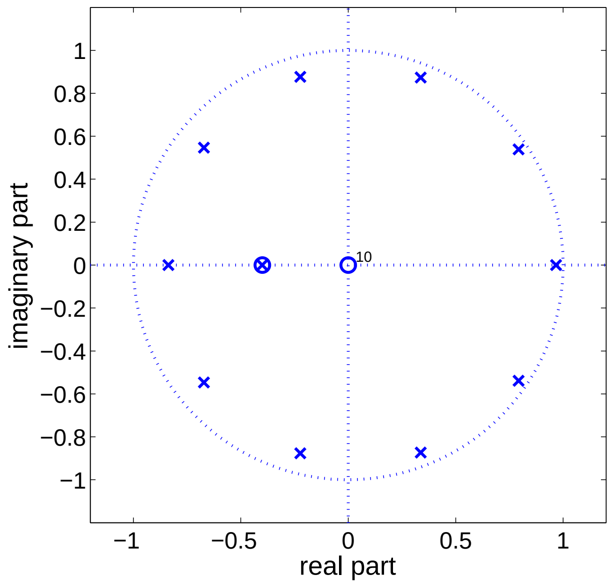


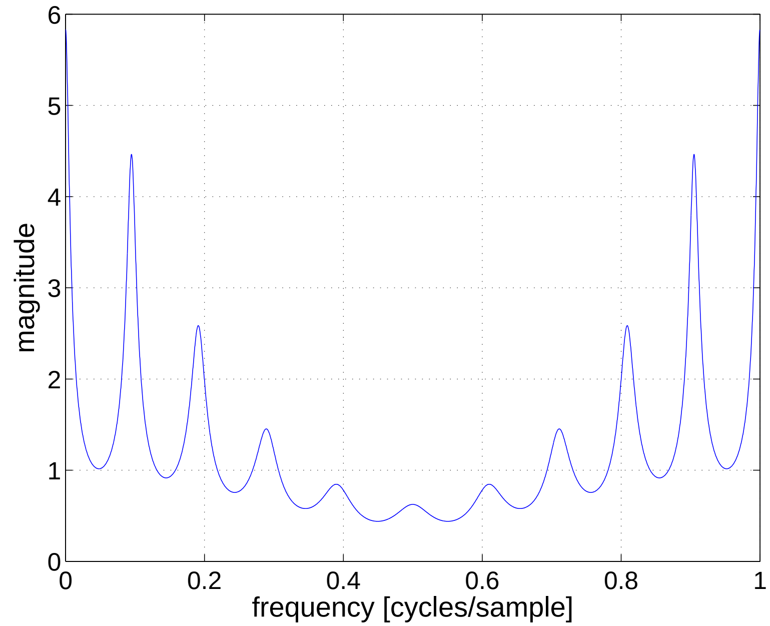


$$H_{LP}(z) = \frac{K(1 - z_1 z^{-1})}{1 - p_1 z^{-1}}$$

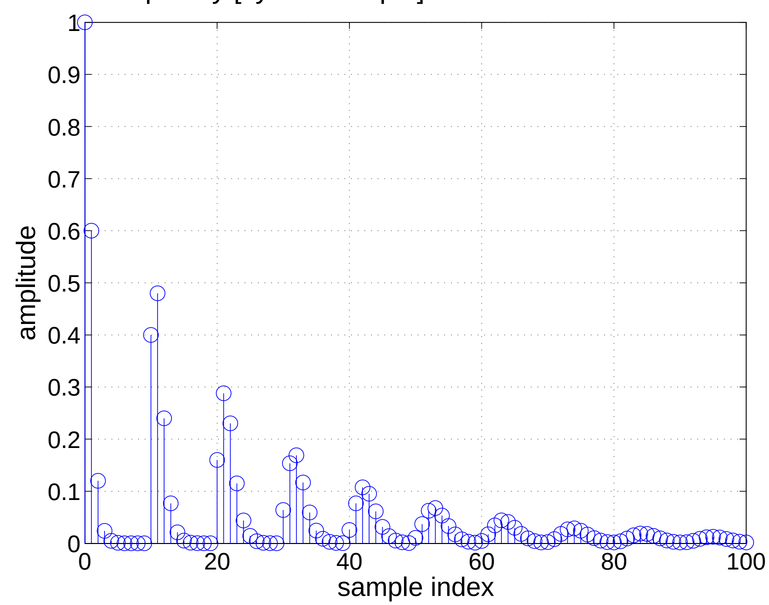
$$z_1 = -0.4, \;\; p_1 = 0.2, \;\; K = 0.5714$$

overall model transfer function





$$H(z) = \frac{1 - p_1 z^{-1}}{1 - p_1 z^{-1} + KAz^{-d} - z_1 KAz^{-(d+1)}}$$



matlab functions used

- `residuez` for partial fraction expansion
- `zplane` for pole zero plot
- `freqz` for frequency response plot
- `filter` for impulse response plot

```
p1 = 0.5;
z1 = -0.5;
K = (1-p1)/(1-z1);
A = 0.7;
d = 10;
num = [1, -z1];
den = [1, -p1, zeros(1,d-2), -K*A, z1*K*A];
impulse = [1,zeros(1,100)]; % unit impulse
zplane(num,den); % poly-zero plot
freqz(num,den);  % frequency response plot
h = filter(num,den,impulse); % impulse response
[R,P,K] = residuez(num,den); % partial fraction expansion
```

all pass filters

- all pass transfer function: (assume B is a real number)

$$H_{AP}(z) = \frac{z^{-1} - B}{1 - Bz^{-1}} = \left(\frac{1}{1 - Bz^{-1}}\right) \cdot \left(z^{-1} - B\right)$$

- this is called an all pass filter because $|H_{AP}(e^{j2\pi f})| = 1$ for $0 \leq f \leq 1$
- define intermediate variable:

$$W(z) = \left(\frac{1}{1 - Bz^{-1}}\right) X(z) \quad \text{or} \quad W(z)(1 - Bz^{-1})X(z)$$

$$Y(z) = \left(z^{-1} - B\right) W(z)$$

this helps in drawing block diagrams (see next page)

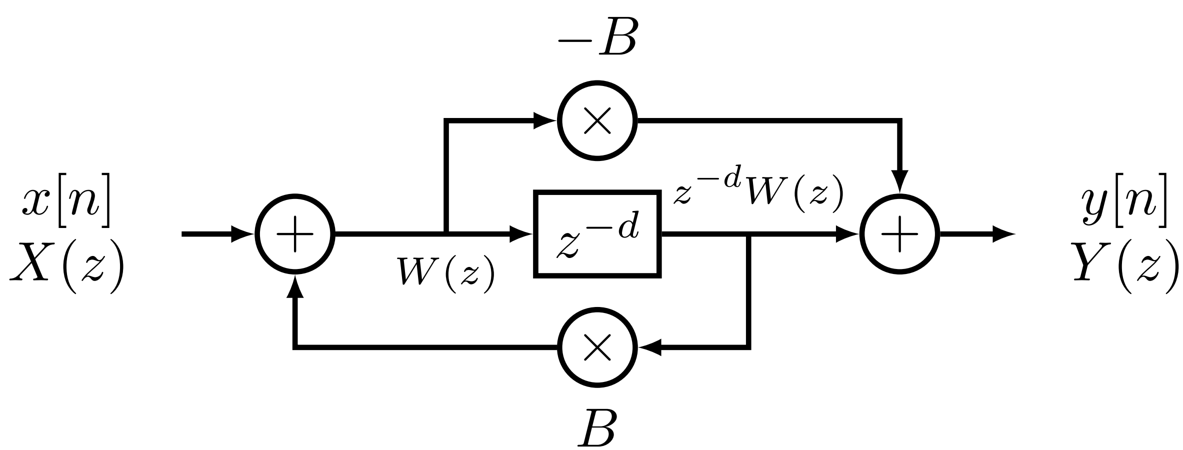
- extend the delay to d samples and get an an all pass reverberator

all pass reverberation filters

- all pass transfer function: (assume B is a real number)

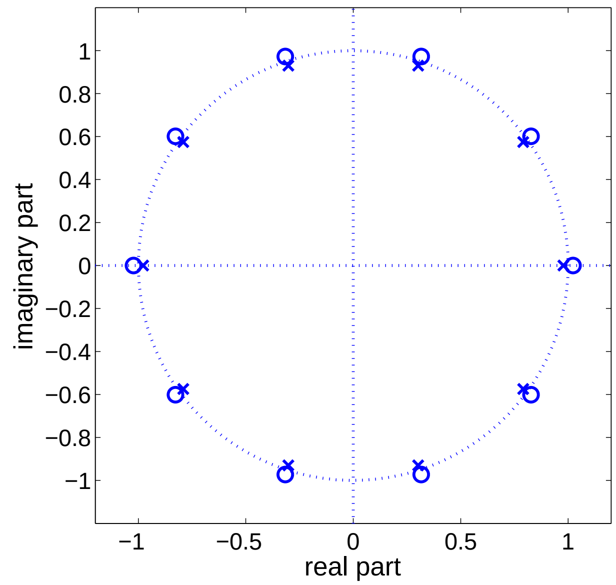
$$H_{APR}(z) = \frac{z^{-d} - B}{1 - Bz^{-d}} = \left(\frac{1}{1 - Bz^{-d}}\right) \cdot \left(z^{-d} - B\right)$$

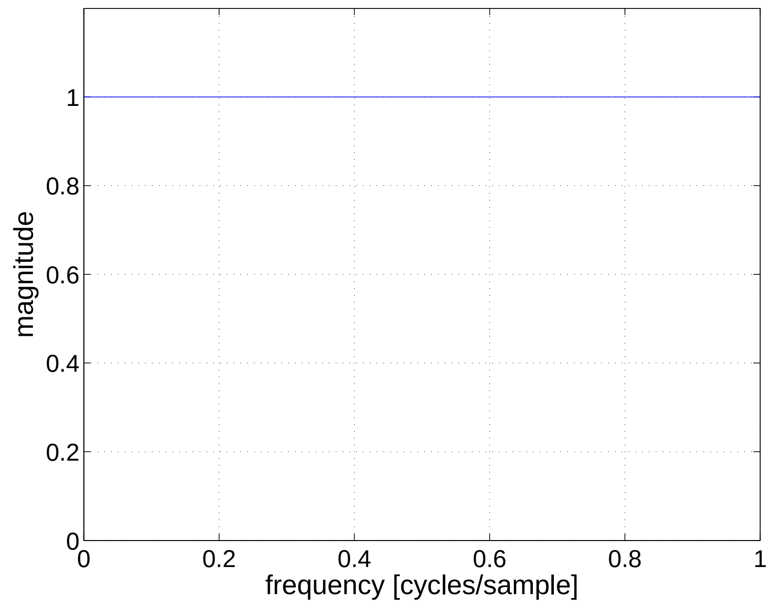
- $|H_{APR}(e^{j2\pi f})| = 1$ for $0 \leq f \leq 1$
- block diagram:



$$W(z)(1 - Bz^{-d}) = X(z), \quad Y(z) = W(z)(z^{-d} - B)$$

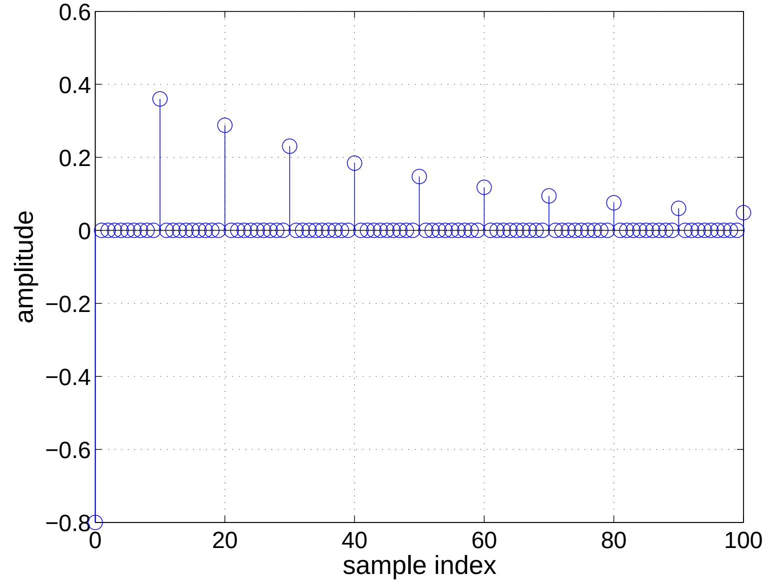
all pass reverberation filter summary





$$H(z) = \frac{z^{-d} - B}{1 - Bz^{-d}}$$

$B = 0.8, \quad d = 10$



requirements

program 1: C $\rightarrow$ speaker and wave file

1. write a C program to add reverberation to an input wave file
2. use the file `woman_singing.wav` (this is a long wave file, process at least 20 seconds of sound)
3. use a series cascade interconnection of three all pass filters having parameters:
   - all pass 1: $B = 0.7, \;\; d = 4551$
   - all pass 2: $B = 0.5, \;\; d = 1237$
   - all pass 3: $B = 0.3, \;\; d = 493$
4. listen to the original and processed wave files and describe the effect of the reverberation filtering