ECE 3640 - Discrete-Time Signals and Systems

Overview of Linear Time-Invariant Systems: Part 2

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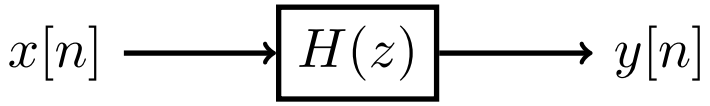
LTI systems and difference equations



$$H(z) = \frac{\sum_{i=0}^{M} b_i z^{-i}}{\sum_{i=0}^{N} a_i z^{-i}}$$

given input $x[n]$ and system parameters $\{a_i\}_{i=0}^{N}$ and $\{b_i\}_{i=0}^{M}$, compute the output $y[n]$ using the difference equation

$$\sum_{i=0}^{N} a_i y[n-i] = \sum_{i=0}^{M} b_i x[n-i], \quad a_0 = 1$$

$$y[n] = \sum_{i=0}^{M} b_i x[n-i] - \sum_{i=1}^{N} a_i y[n-i]$$

usually set initial conditions to zero $y[n-1] = y[n-2] = \cdots = y[n-N] = 0$

LTI system example: performance hall

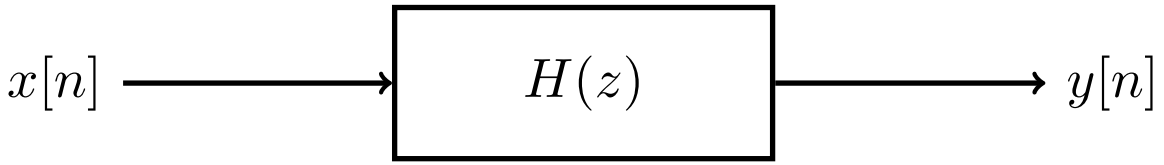
- performance hall is an acoustic LTI system
- admits a difference equation model
- difference equation model is indistinguishable from the real hall
- don't need the real hall; can simulate the real hall

LTI system example: performance hall

A performance hall is actually many LTI systems depending on the location of the excitation (instrument, speaker, mouth, etc.) and the location of the measurement (microphone, ear, etc.).

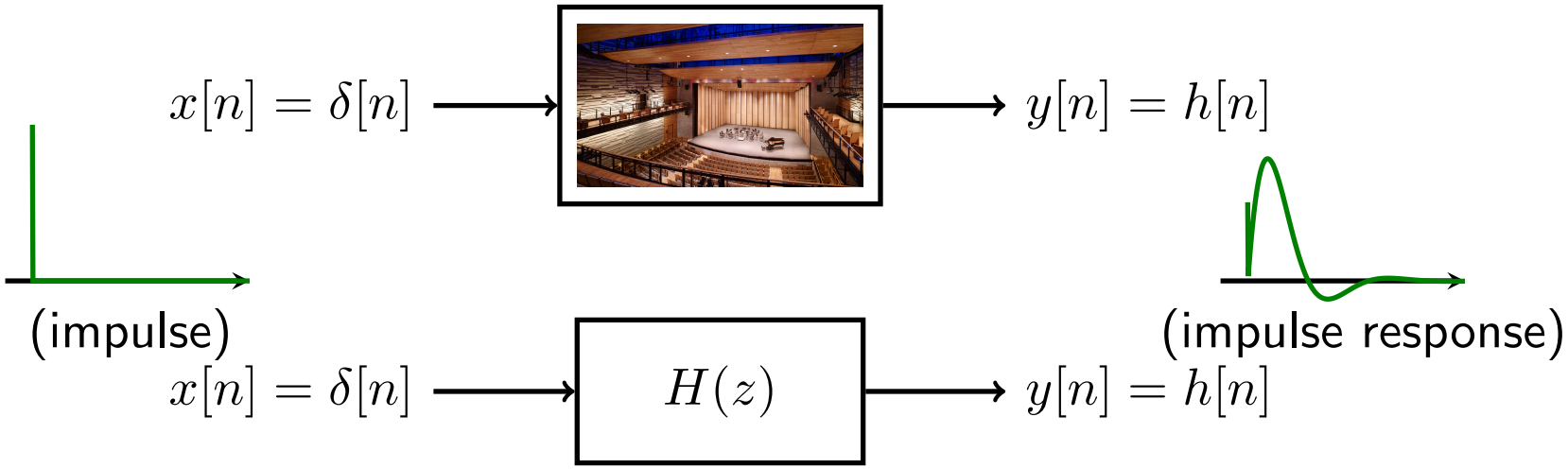
The response of the hall varies with air temperature, audience presence, etc.

performance hall is LTI system



- these two have the same input-output behavior
- expensive to use the performance hall
- the LTI system is software (inexpensive)
- Q: what would my music sound like if played in a performance hall
- A: run music through the difference equation

impulse response



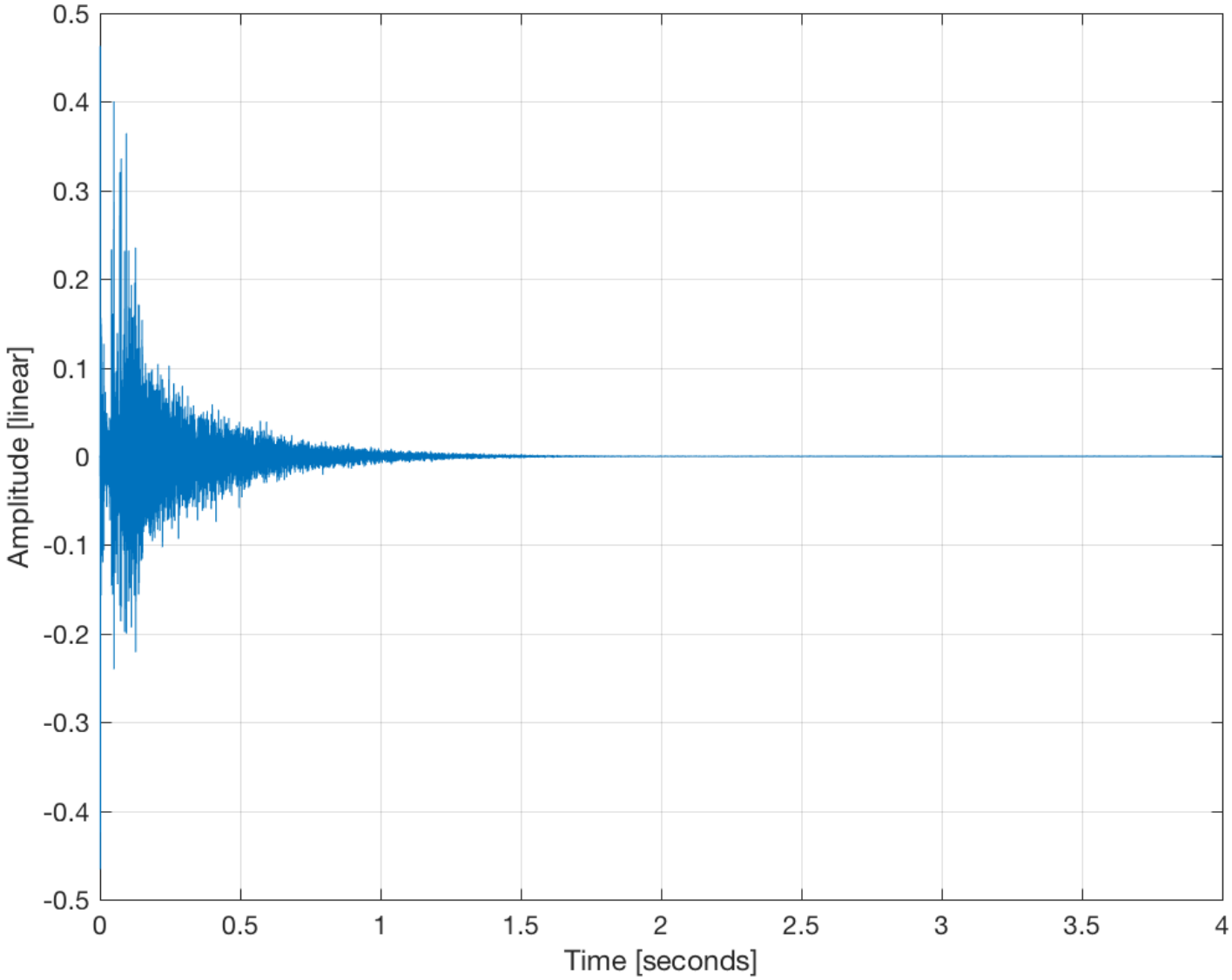
- do not know $\{a_i\}_{i=0}^{N}$ or $\{b_i\}_{i=0}^{M}$
- measure impulse response $h[n]$ and convolve $y[n] = h[n] * x[n]$

$$y[n] = \sum_{k=0}^{\infty} h[k]x[n-k] = \sum_{i=0}^{M} b_i x[n-i] - \sum_{i=1}^{N} a_i y[n-i]$$

- convolution with $h[n]$ gives same answer as difference equation
- in general $h[n]$ has infinite length
- in practice truncate to finite length

measured impulse response

measured impulse response



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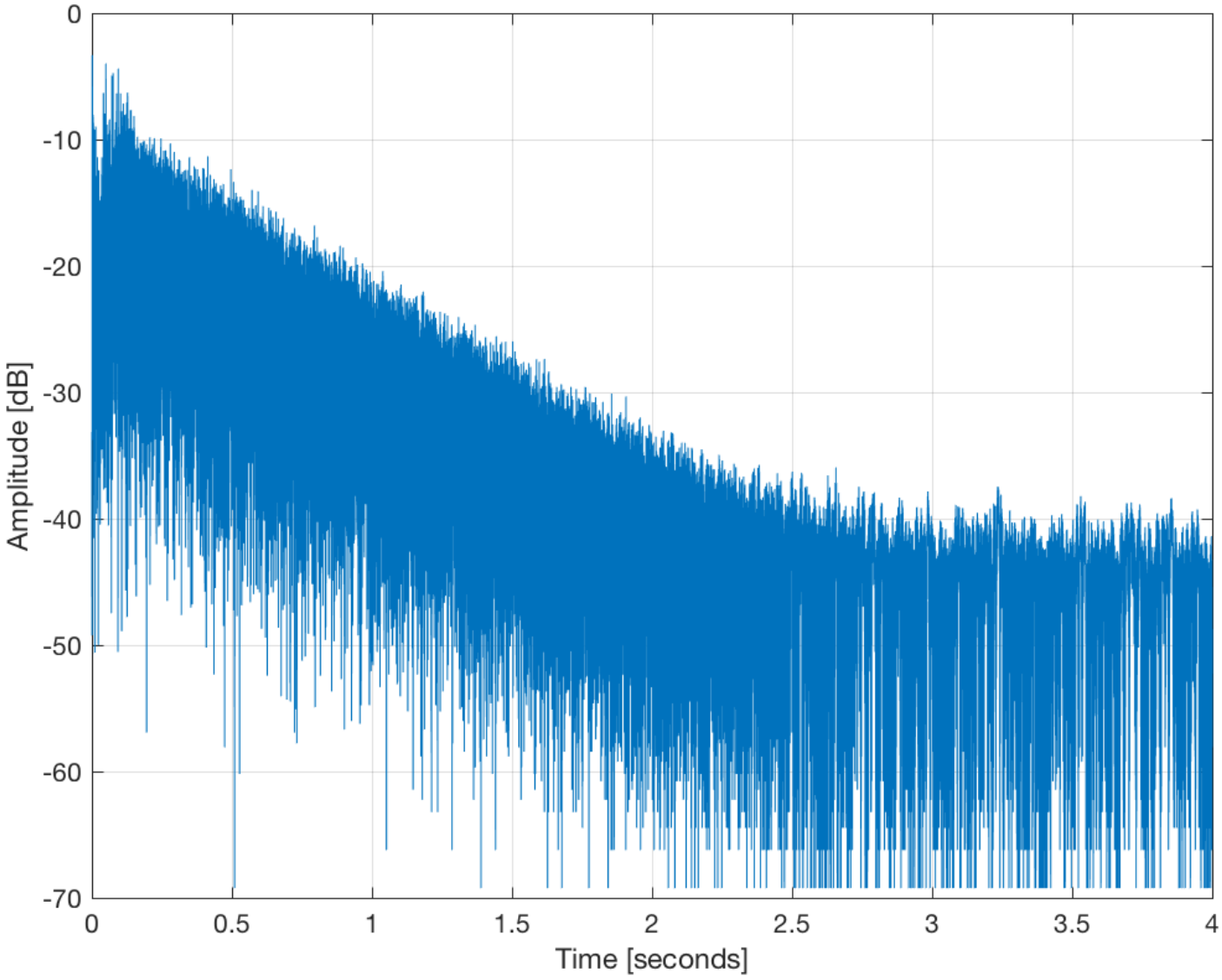
measuring impulse response



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enclosure simulation

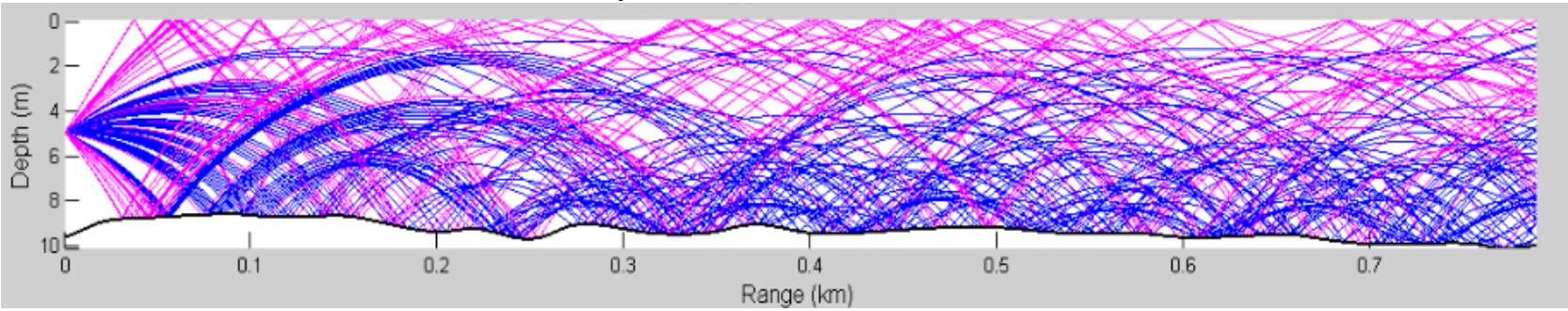
- Step 1. Measure impulse response in an "enclosure" such as a hallway, stairwell, racquetball court, basketball court, etc. using an impulse such as clap, blocks of wood, balloon pop, cap gun, snap, etc.

- Step 2. Convolve impulse response with speech or music (make sure sample rates match)

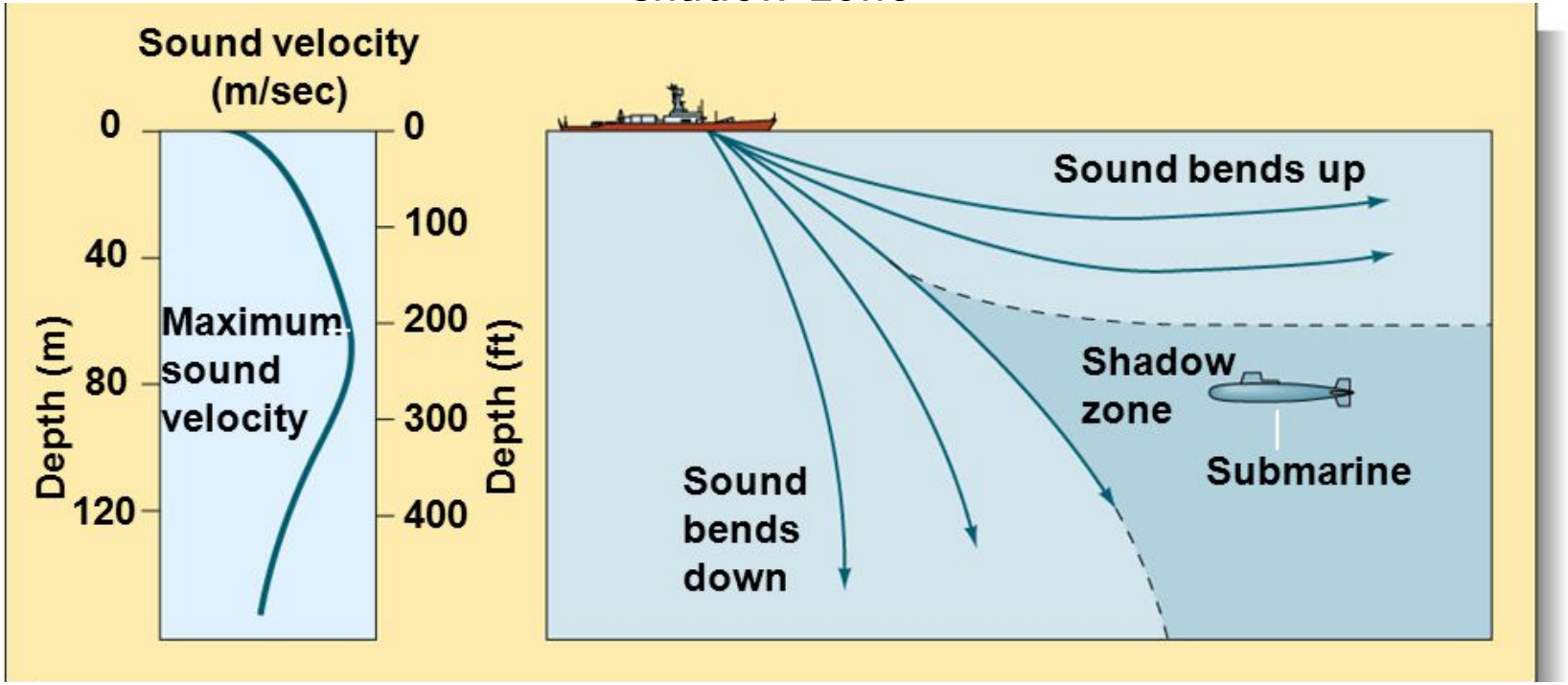
- Step 3. Need efficient filtering algorithm

underwater acoustic impulse response considerations

sound paths in shallow water

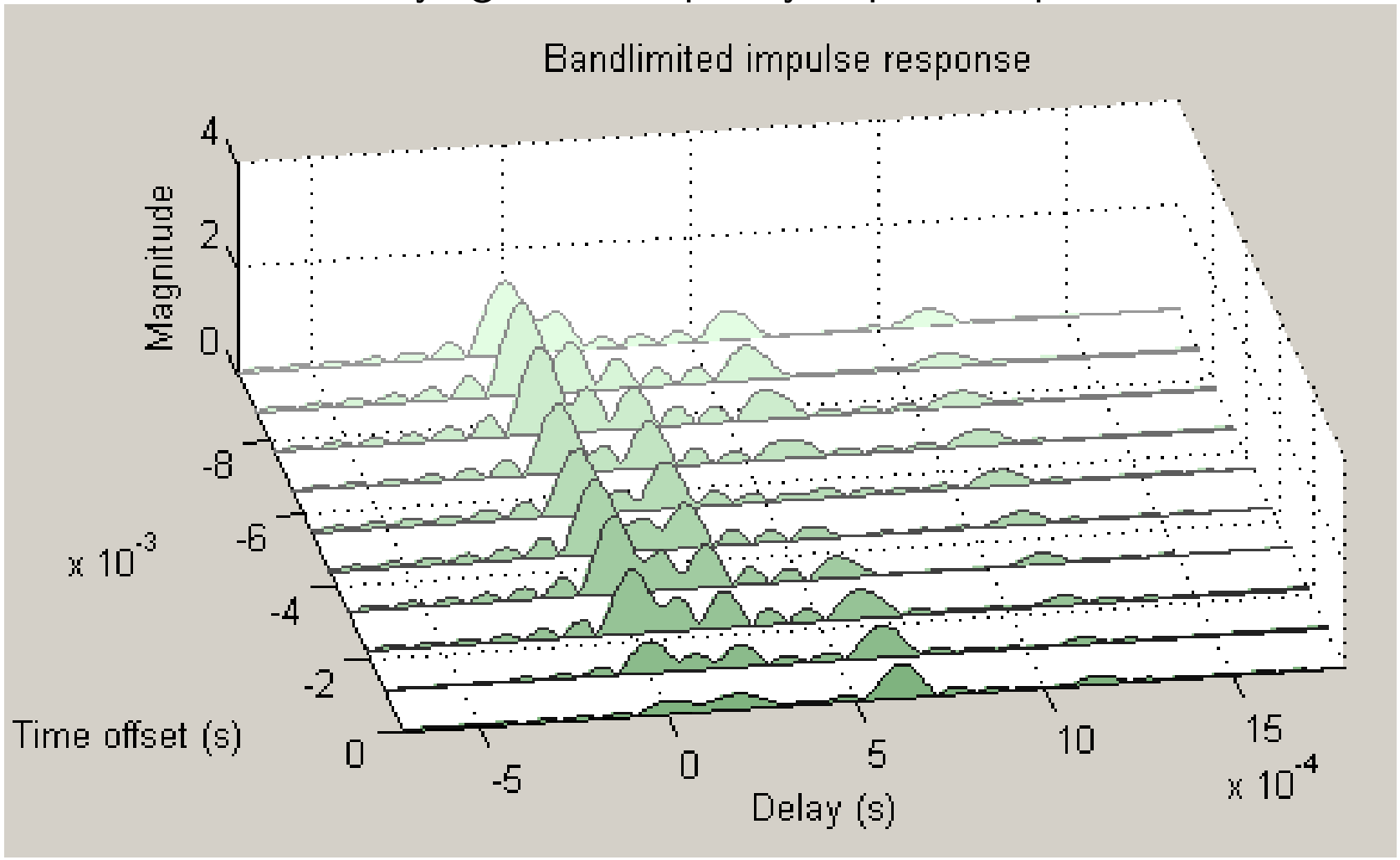


shadow zone

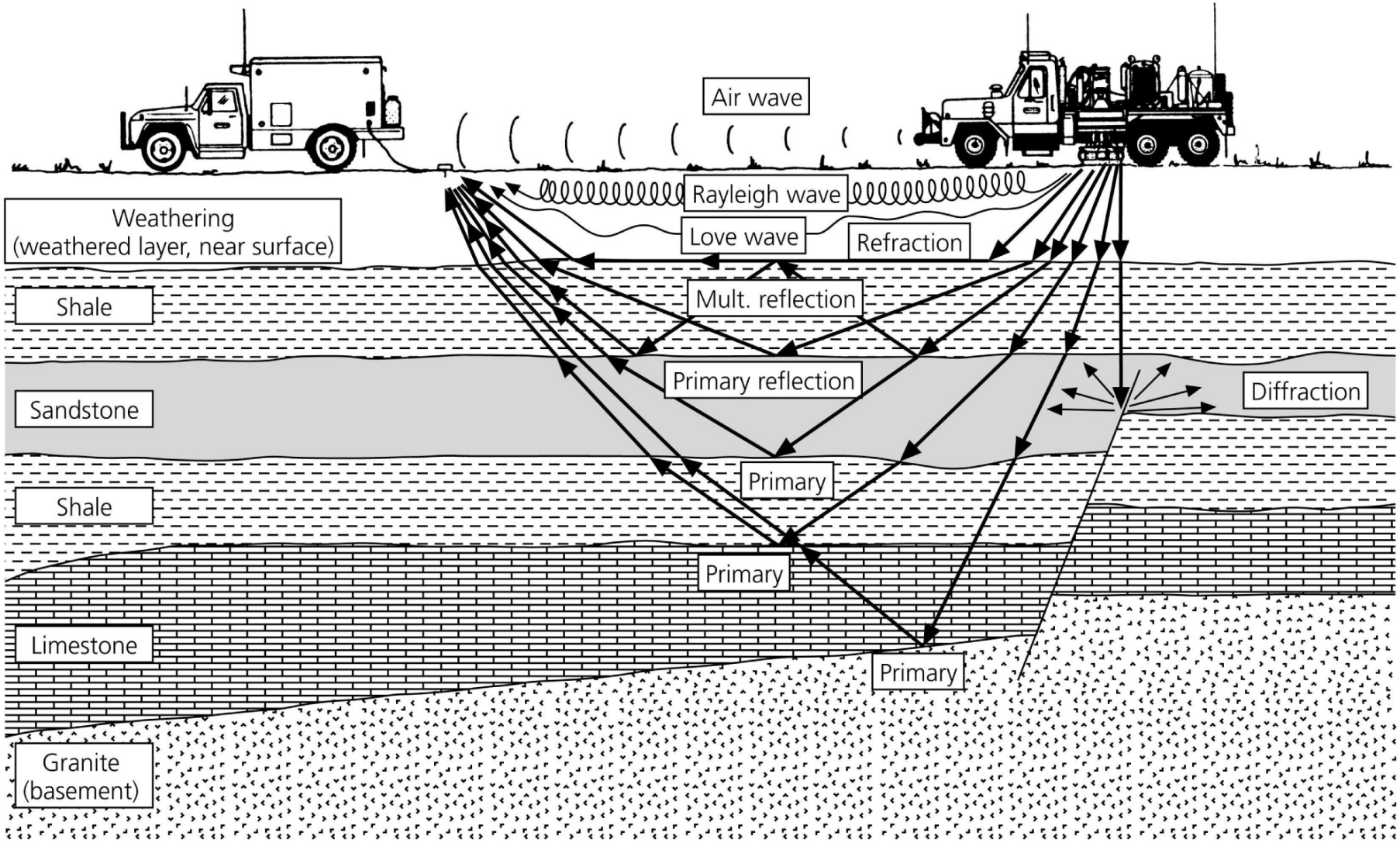


radio frequency impulse response considerations

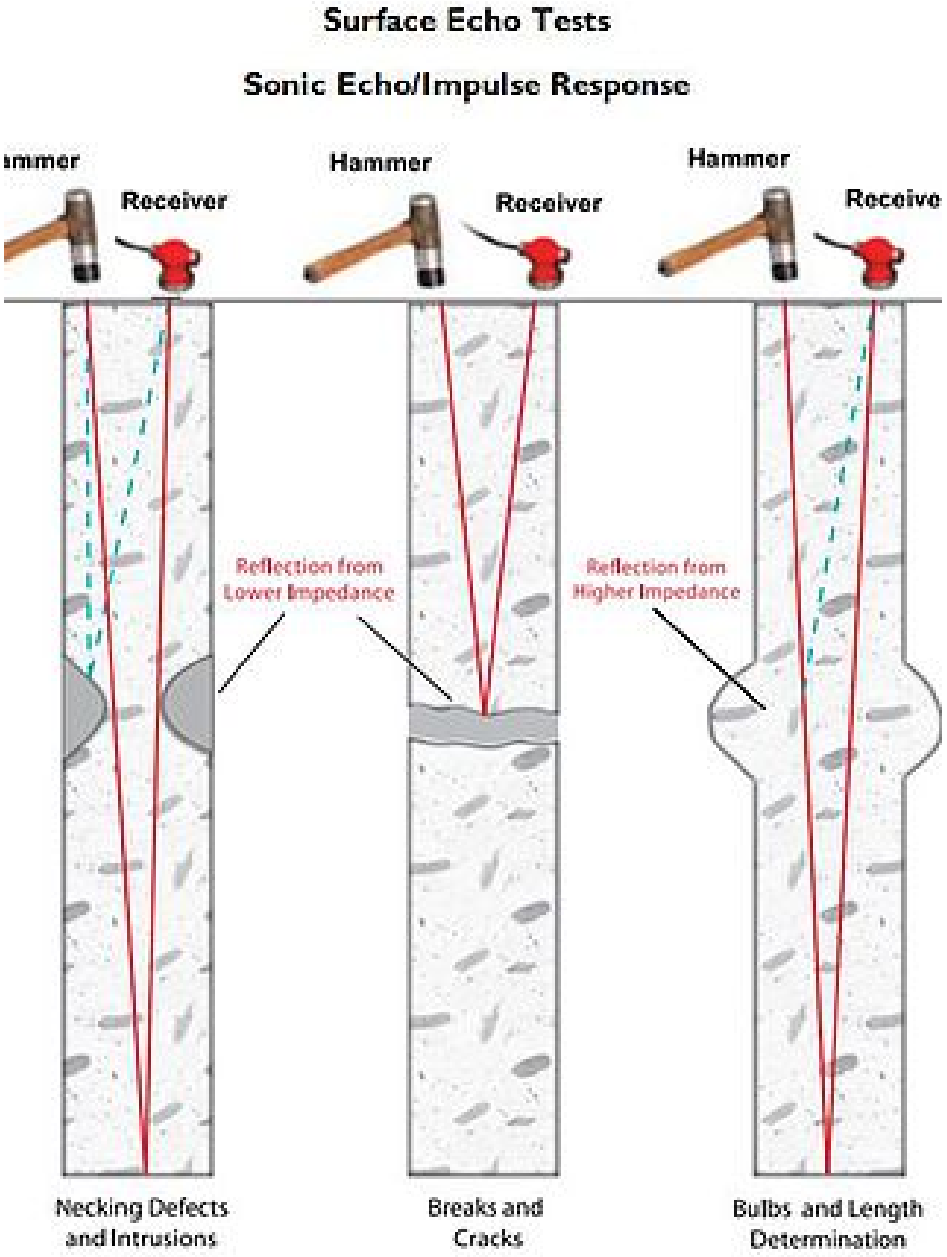
time-varying radio frequency impulse response



impulse response for geophysical exploration



near-surface impulse response kit



2D impulse response & 2D convolution

original image

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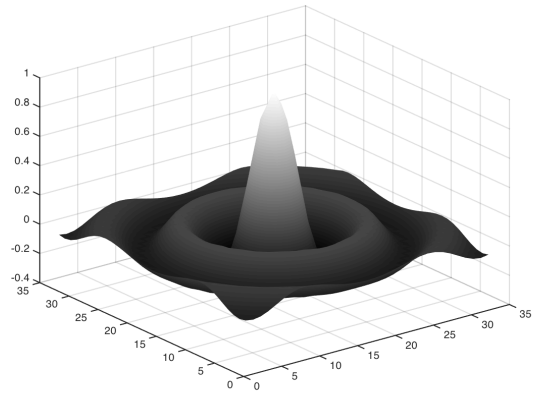

point spread function
(impulse response)

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blurred image

- 2D convolution: $y[m,n] = \sum_{i=0}^{N} \sum_{j=0}^{N} h[i,j]x[m-i,n-j]$
- problem: given blurry image, recover the original
- applications in: radio astronomy, medical imaging, computational photography, microscopy, etc.