Convolution Lab

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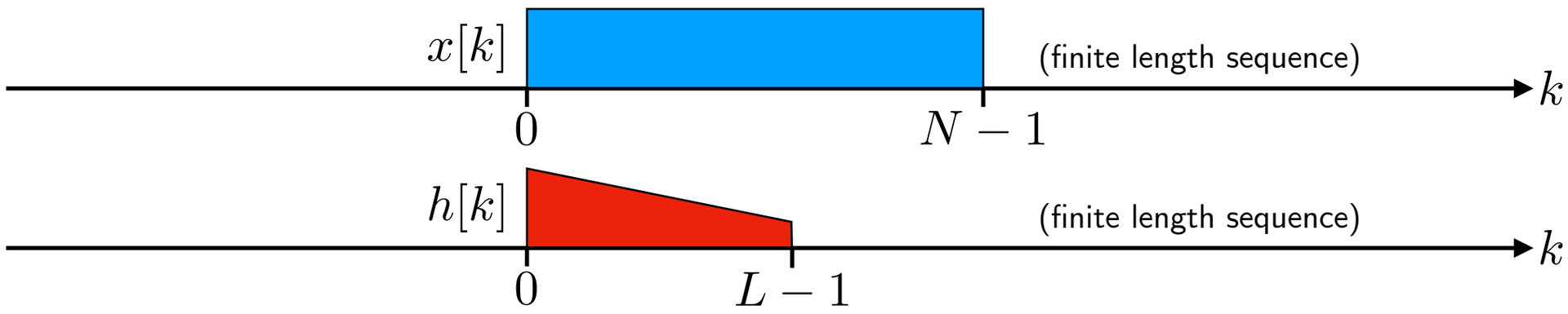
Definition of Convolution

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k], \qquad -\infty < n < \infty$$

(convolution for infinite length sequences)



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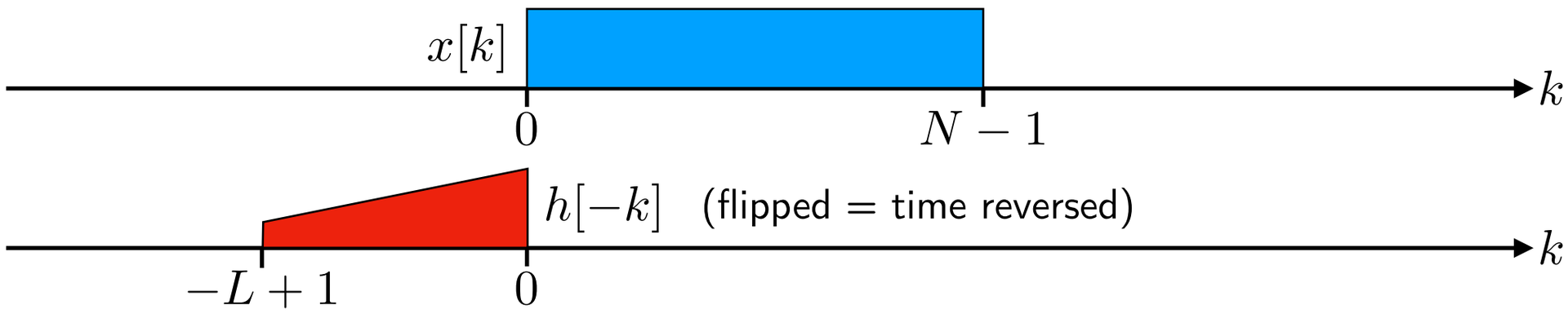

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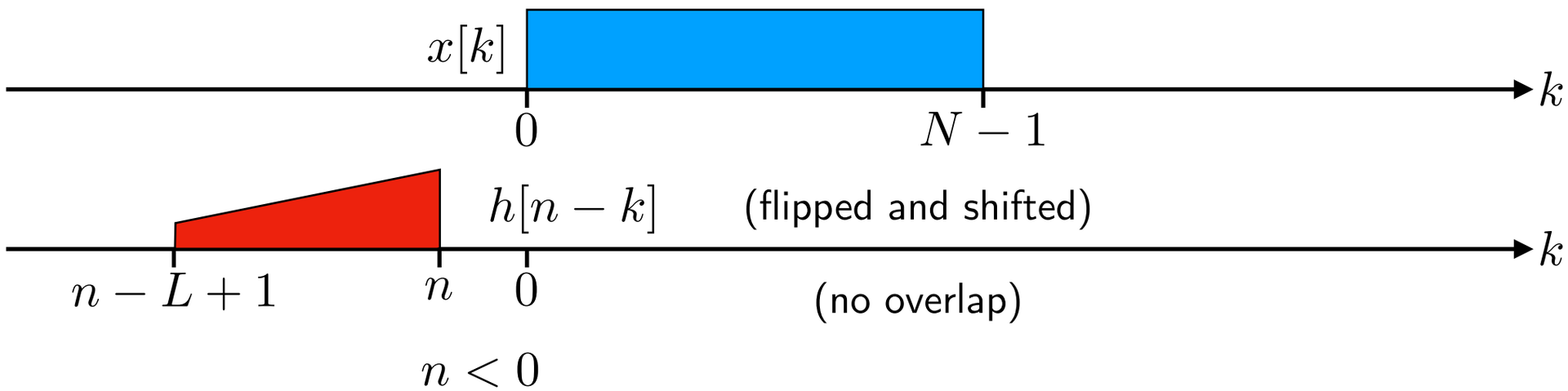

$$y[n] = 0, \quad n < 0$$

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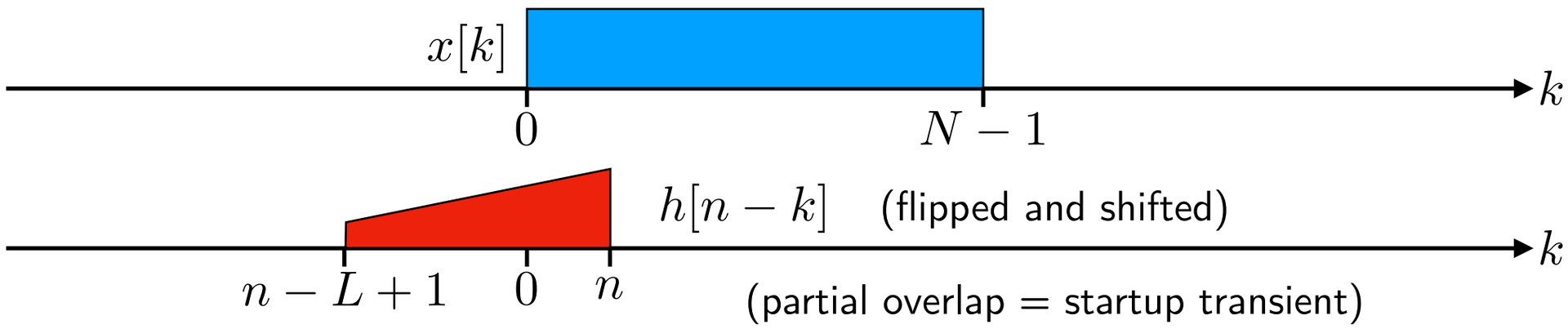

$$n - L + 1 < 0 \leq n \quad \Rightarrow \quad 0 \leq n < L - 1$$

$$0 \leq n \leq L - 2 < L - 1$$

$$y[n] = \sum_{k=0}^{n} x[k]h[n-k], \quad 0 \leq n \leq L-2$$

Number of samples in startup transient = L-1

$$(L-2) - (0) + 1 = L - 1$$

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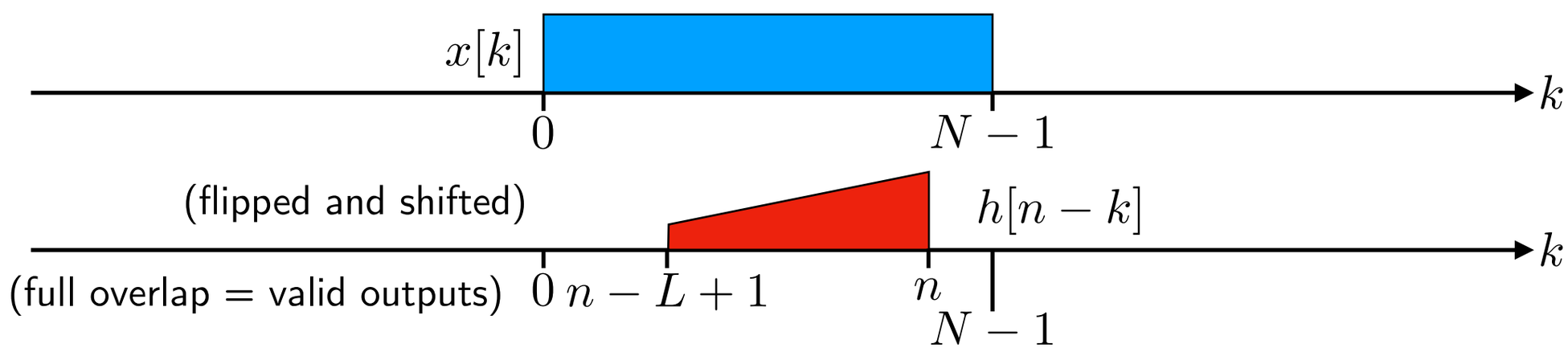

$$0 \le n-L+1 \quad \text{and} \quad n \le N-1 \quad \Rightarrow \quad L-1 \le n \le N-1$$

$$y[n] = \sum_{k=n-L+1}^{n} x[k]h[n-k], \quad L-1 \le n \le N-1$$

Number of valid output samples = N - L + 1

$$(N-1)-(L-1)+1 = N-L+1$$

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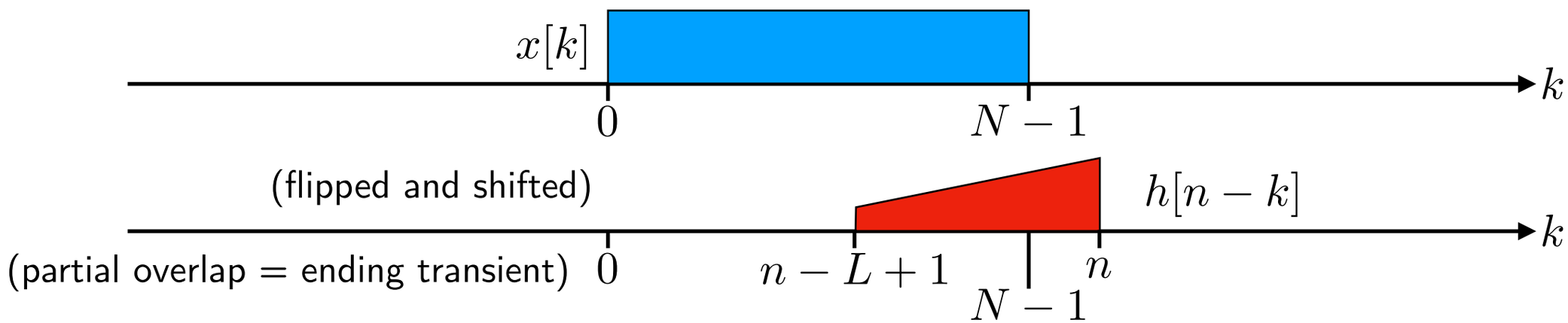
$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k], \qquad -\infty < n < \infty$$



$$n - L + 1 \leq N - 1 < n \quad \Rightarrow \quad N - 1 < n \leq N + L - 2$$

$$N - 1 < N \leq n \leq N + L - 2$$

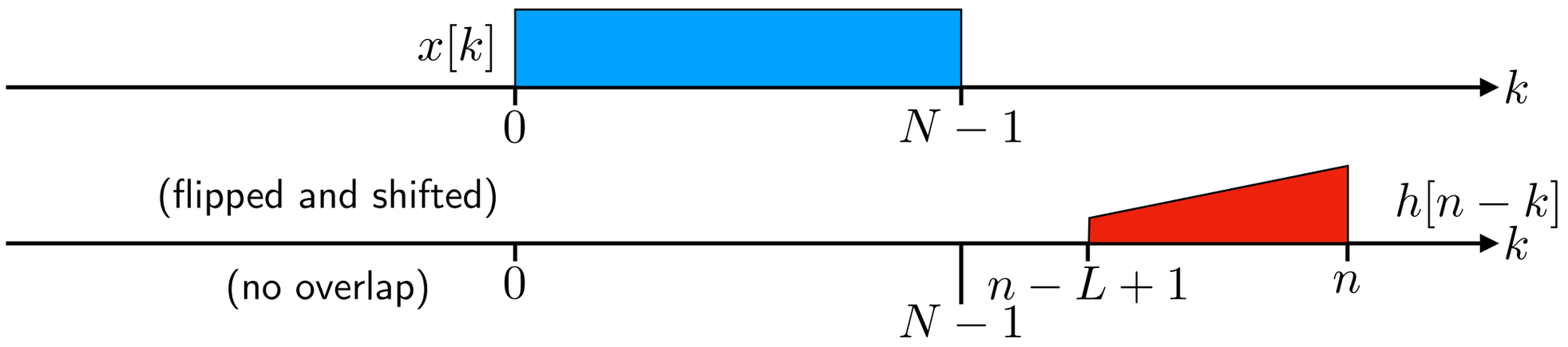
$$y[n] = \sum_{k=n-L+1}^{N-1} x[k]h[n-k], \quad N - 1 < n \leq N + L - 2$$

Number of samples in ending transient = L-1

$$(N + L - 2) - (N) + 1 = L - 1$$

Definition of Convolution

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k], \qquad -\infty < n < \infty$$



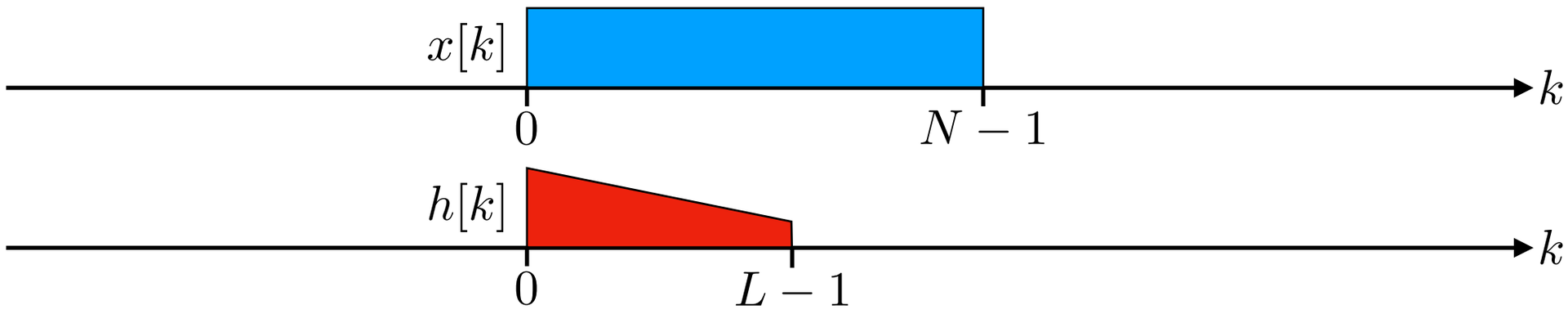
$$n - L + 1 > N - 1 \quad \Rightarrow \quad n > N + L - 2$$

$$y[n] = 0, \quad n > N + L - 2$$

Definition of Convolution

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k], \qquad -\infty < n < \infty$$

(convolution for infinite length sequences)



$$y[n] = \sum_{k=\max(0,n-L+1)}^{\min(N-1,n)} x[k]h[n-k], \quad 0 \le n \le N+L-2$$

(convolution for finite length sequences)
Total length = N+L-1

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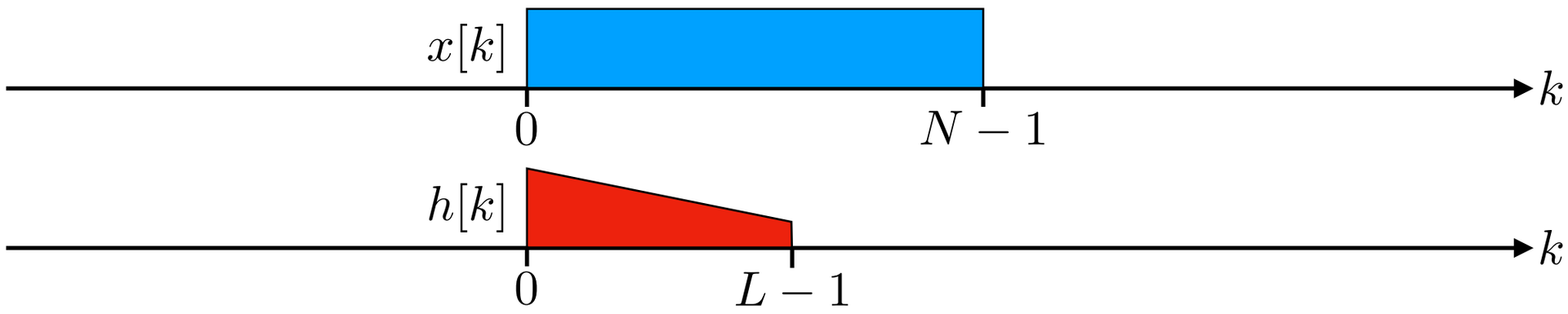
(convolution for infinite length sequences)



$$y[n] = \sum_{k=\max(0,n-L+1)}^{\min(N-1,n)} x[k]h[n-k], \quad 0 \le n \le N+L-2$$

(convolution for finite length sequences)
Total length = N+L-1

How do we program this on a computer?

Mechanics of Convolution

$$y[n] = \sum_{k=\max(0,n-L+1)}^{\min(N-1,n)} x[k]h[n-k], \quad 0 \le n \le N+L-2$$

	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}
	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7				
	$\leftarrow \quad L-1 \quad \rightarrow$			$\leftarrow \quad N \quad \rightarrow$					$\leftarrow \quad L-1 \quad \rightarrow$			

$L = 5$

$N = 8$

$N + L - 1 = 12$

Mechanics of Convolution

$$y[n] = \sum_{k=\max(0,n-L+1)}^{\min(N-1,n)} x[k]h[n-k], \quad 0 \le n \le N+L-2$$

				y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}
				x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7				
h_4	h_3	h_2	h_1	h_0											
	h_4	h_3	h_2	h_1	h_0										
		h_4	h_3	h_2	h_1	h_0									
			h_4	h_3	h_2	h_1	h_0								
				h_4	h_3	h_2	h_1	h_0							
					h_4	h_3	h_2	h_1	h_0						
						h_4	h_3	h_2	h_1	h_0					
							h_4	h_3	h_2	h_1	h_0				
								h_4	h_3	h_2	h_1	h_0			
									h_4	h_3	h_2	h_1	h_0		
										h_4	h_3	h_2	h_1	h_0	
											h_4	h_3	h_2	h_1	h_0
$\leftarrow$	$L-1$	$\rightarrow$		$\leftarrow$			N				$\rightarrow$	$\leftarrow$		$L-1$	$\rightarrow$

(all shifts h[n-k] are shown)

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Mechanics of Convolution

$$y[n] = \sum_{k=\max(0,n-L+1)}^{\min(N-1,n)} x[k]h[n-k], \quad 0 \le n \le N+L-2$$

				y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}
				x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7				
h_4	h_3	h_2	h_1	h_0											
	h_4	h_3	h_2	h_1	h_0										
		h_4	h_3	h_2	h_1	h_0									
			h_4	h_3	h_2	h_1	h_0								
				h_4	h_3	h_2	h_1	h_0							
					h_4	h_3	h_2	h_1	h_0						
						h_4	h_3	h_2	h_1	h_0					
							h_4	h_3	h_2	h_1	h_0				
								h_4	h_3	h_2	h_1	h_0			
									h_4	h_3	h_2	h_1	h_0		
										h_4	h_3	h_2	h_1	h_0	
											h_4	h_3	h_2	h_1	h_0
$\leftarrow$	$L-1$	$\rightarrow$		$\leftarrow$			N				$\rightarrow$	$\leftarrow$		$L-1$	$\rightarrow$

time reversed order, h[n-k]

natural order, x[k]

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Mechanics of Convolution

$$y[n] = \sum_{k=\max(0,n-L+1)}^{\min(N-1,n)} x[k]h[n-k], \quad 0 \le n \le N+L-2$$

(startup transient) (valid outputs) (ending transient)

				y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}
				x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7				
h_4	h_3	h_2	h_1	h_0											
	h_4	h_3	h_2	h_1	h_0										
		h_4	h_3	h_2	h_1	h_0									
$L-1$ (partial overlap)			h_4	h_3	h_2	h_1	h_0								
				h_4	h_3	h_2	h_1	h_0							
					h_4	h_3	h_2	h_1	h_0						
				$N-L+1$ (full overlap)			h_4	h_3	h_2	h_1	h_0				
								h_4	h_3	h_2	h_1	h_0			
									h_4	h_3	h_2	h_1	h_0		
										h_4	h_3	h_2	h_1	h_0	
											h_4	h_3	h_2	h_1	h_0
$\leftarrow L-1 \rightarrow$				$\leftarrow N \rightarrow$								$\leftarrow L-1 \rightarrow$			

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$$y[n] = \sum_{k=\max(0,n-L+1)}^{\min(N-1,n)} x[k]h[n-k], \quad 0 \le n \le N+L-2$$

				y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}
				x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7				
h_4	h_3	h_2	h_1	h_0											
	h_4	h_3	h_2	h_1	h_0										
		h_4	h_3	h_2	h_1	h_0									
			h_4	h_3	h_2	h_1	h_0								
				h_4	h_3	h_2	h_1	h_0							
					h_4	h_3	h_2	h_1	h_0						
						h_4	h_3	h_2	h_1	h_0					
							h_4	h_3	h_2	h_1	h_0				
								h_4	h_3	h_2	h_1	h_0			
									h_4	h_3	h_2	h_1	h_0		
										h_4	h_3	h_2	h_1	h_0	
											h_4	h_3	h_2	h_1	h_0
$\leftarrow L-1 \rightarrow$				$\leftarrow N \rightarrow$								$\leftarrow L-1 \rightarrow$			

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Mechanics: (1) flip h[-k], (2) shift h[n-k], (3) multiply x[k]h[n-k], (4) add

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$$y[n] = \sum_{k=\max(0,n-L+1)}^{\min(N-1,n)} x[k]h[n-k], \quad 0 \le n \le N+L-2$$

				y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}
0	0	0	0	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	0	0	0	0
h_4	h_3	h_2	h_1	h_0											
	h_4	h_3	h_2	h_1	h_0										
		h_4	h_3	h_2	h_1	h_0									
			h_4	h_3	h_2	h_1	h_0								
				h_4	h_3	h_2	h_1	h_0							
					h_4	h_3	h_2	h_1	h_0						
						h_4	h_3	h_2	h_1	h_0					
							h_4	h_3	h_2	h_1	h_0				
								h_4	h_3	h_2	h_1	h_0			
									h_4	h_3	h_2	h_1	h_0		
										h_4	h_3	h_2	h_1	h_0	
											h_4	h_3	h_2	h_1	h_0
$\leftarrow$	$L-1$	$\rightarrow$		$\leftarrow$			N				$\rightarrow$	$\leftarrow$		$L-1$	$\rightarrow$

Zero padding the input x[n] makes coding convolution much easier.

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Mechanics: (1) flip h[-k], (2) shift h[n-k], (3) multiply x[k]h[n-k], (4) add

Implementation of Convolution

Goto Matlab