HW 3 LTI Systems

1. Let the impulse response $h[n]$ be given by the piece-wise defined sequence

$$h[n] = \begin{cases} 1, & -3 \le n \le 3, \\ 0, & \text{otherwise} \end{cases}$$

Convolve $h[n]$ with the following input signals. In each case, write the convolution result $y[n] = h[n] * x[n]$ as a piece-wise defined sequence.

(a) $x_1[n] = a^{|n|}$, where $|a| < 1$
(b) $x_2[n] = a^n u[n]$, where $|a| < 1$
(c) $x_3[n] = a^n (u[n] - u[n-10])$
(d) $x_4[n] = x_2[n] + 3x_2[n-10]$ (hint: use linearity and time invariance)
(e) $x_5[n] = e^{j2\pi fn}$ (everlasting complex exponential) and evaluate the output when $f = \frac{3}{7}$ and when $f = \frac{5}{14}$.
(f) $x_6[n] = e^{j2\pi fn} u[n]$ (causal complex exponential) and evaluate the output when $f = \frac{3}{7}$
(g) Is the system with impulse response $h[n]$ causal?
(h) Is the system with impulse response $h[n]$ stable?

2. Let $y[n] = x[n] * x^*[-n]$. Give an interpretation of $y[0]$.

3. Let $h[n]$ be given by

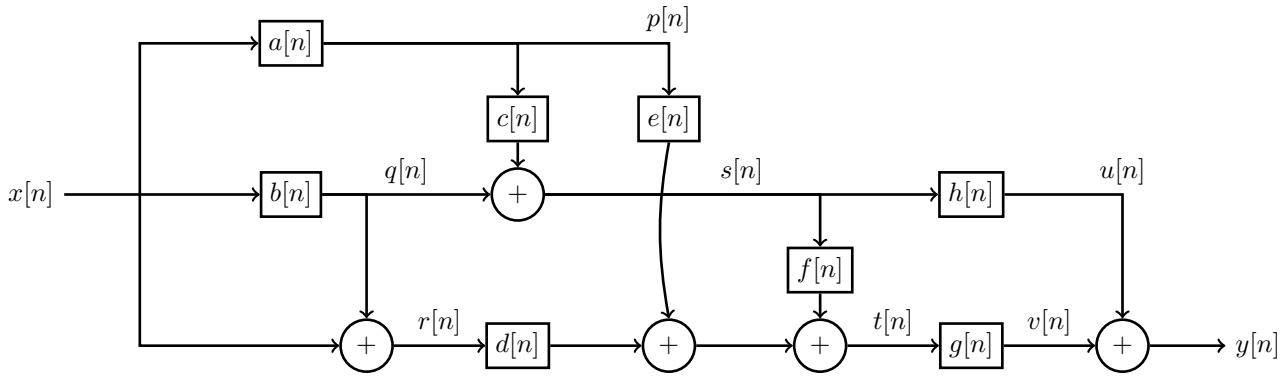
$$\begin{aligned} h[n] &= 0, \quad n \le -3 \\ h[-2] &= 1 \\ h[-1] &= 2 \\ h[n] &= 3, \quad n = 0, 1, 2, 3, 4, 5 \\ h[n] &= 0, \quad n \ge 6 \end{aligned}$$

Let $x[n] = 0.9^n u[n]$. Do the following:

(a) Sketch $h[-k]$ on the k axis
(b) Sketch $h[n-k]$ on the k axis
(c) Sketch $h[10-k]$ on the k axis
(d) Sketch $x[k]$ on the k axis
(e) Compute the value of $y[10]$.

In your sketches, include the region $k = -2, -1, 0, \cdots, 15$.

4. Consider the cascade interconnection of LTI systems show below. Find an expression for the impulse response of an equivalent system.



5. Let $h[0], h[1], \cdots, h[L-1]$ be a length L impulse response and let $x[0], x[1], \cdots, x[M-1]$ be a length $M > L$ input sequence. Let $y[0], y[1], \cdots, y[N-1]$ be the length N convolution result.

   (a) What is the length N of the output $y[n]$ in terms of L and M?

   (b) How many samples of the output $y[n]$ (and which ones) are starting transients?

   (c) How many samples of the output $y[n]$ (and which ones) are ending transients?

   (d) How many samples of the output $y[n]$ (and which ones) are valid output samples.

6. Sketch the impulse response for the equivalent system.