ECE 3640 - Discrete-Time Signals and Systems
Midterm - Spring 2024

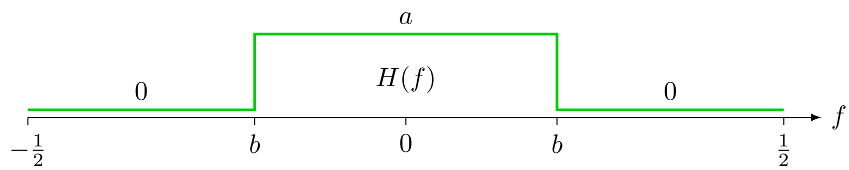
Name:	Due: Tuesday, 19 March 2023 at 11:59 PM on Canvas.

Instructions:

1. Allowed resources: Your text book, your homework, your notes, the course web site, and Matlab.
2. Do not talk to anyone about this exam or get help from any source on this exam.
3. Write your answers in the spaces provided. Draw a box around your answers.
4. By signing in the name box above, you verify that you have complied with these instructions.

1. Follow the steps below.

   (a) Suppose $h[n] = \sin(n)/n$ is the impulse response of a system with frequency response $H(f)$ shown below. Determine the values of a and b.



$a =$

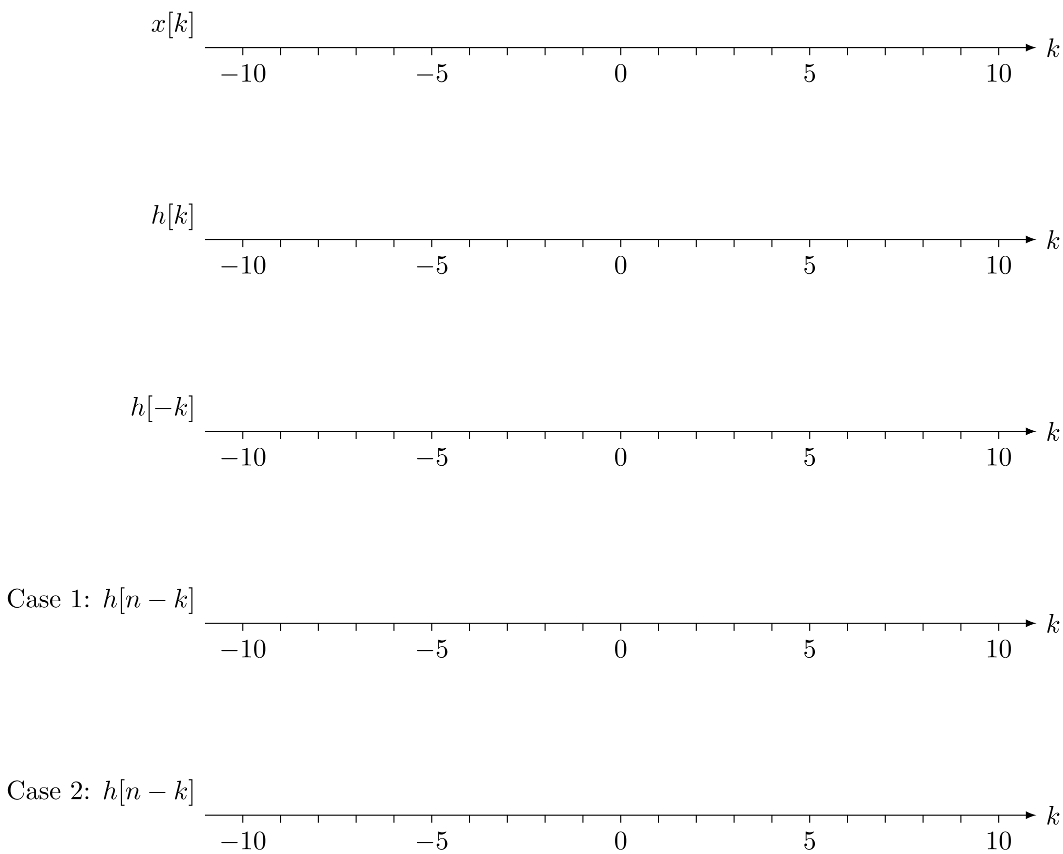
$b =$

   (b) Let $y[n]$ be the output of this system when the input is $x[n] = \sin(n)/n$. Compute $y[n]$.

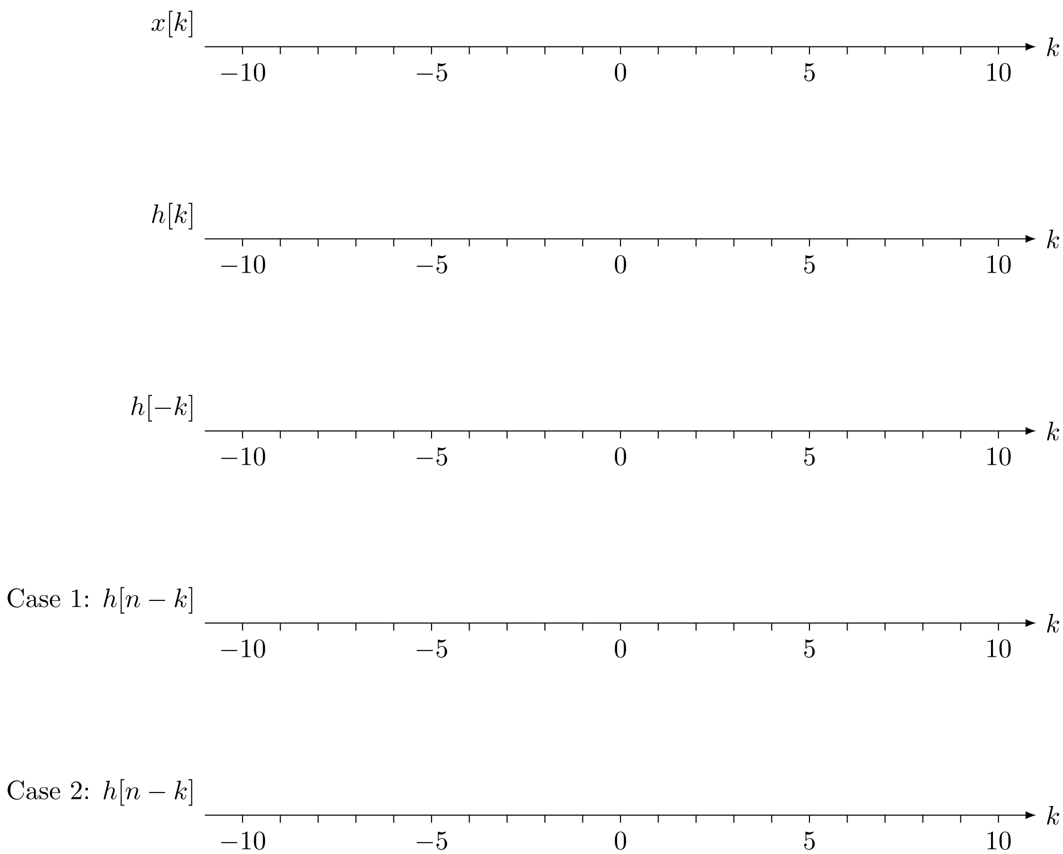
$y[n] =$

2. Follow the steps below. Assume that $|a| < 1$ and $|b| < 1$ in this problem.

(a) Let $x[n] = a^n u[n]$ and $h[n] = b^n u[n]$. What is $y[n] = x[n] * h[n] = \sum_k x[k]h[n-k]$? Write $y[n]$ as a piecewise defined function. Sketch the signals involved in the convolution indicated on the axes below. Case 1 is the "no overlap" case and Case 2 is the "partial overlap" case.



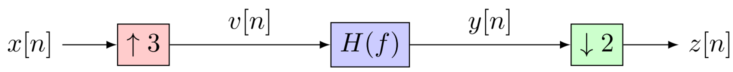
(b) Let $x[n] = a^n u[n-5]$ and $h[n] = b^n u[n+5]$. What is $y[n] = x[n] * h[n]$? Sketch the signals and write $y[n]$ as a piecewise defined function.

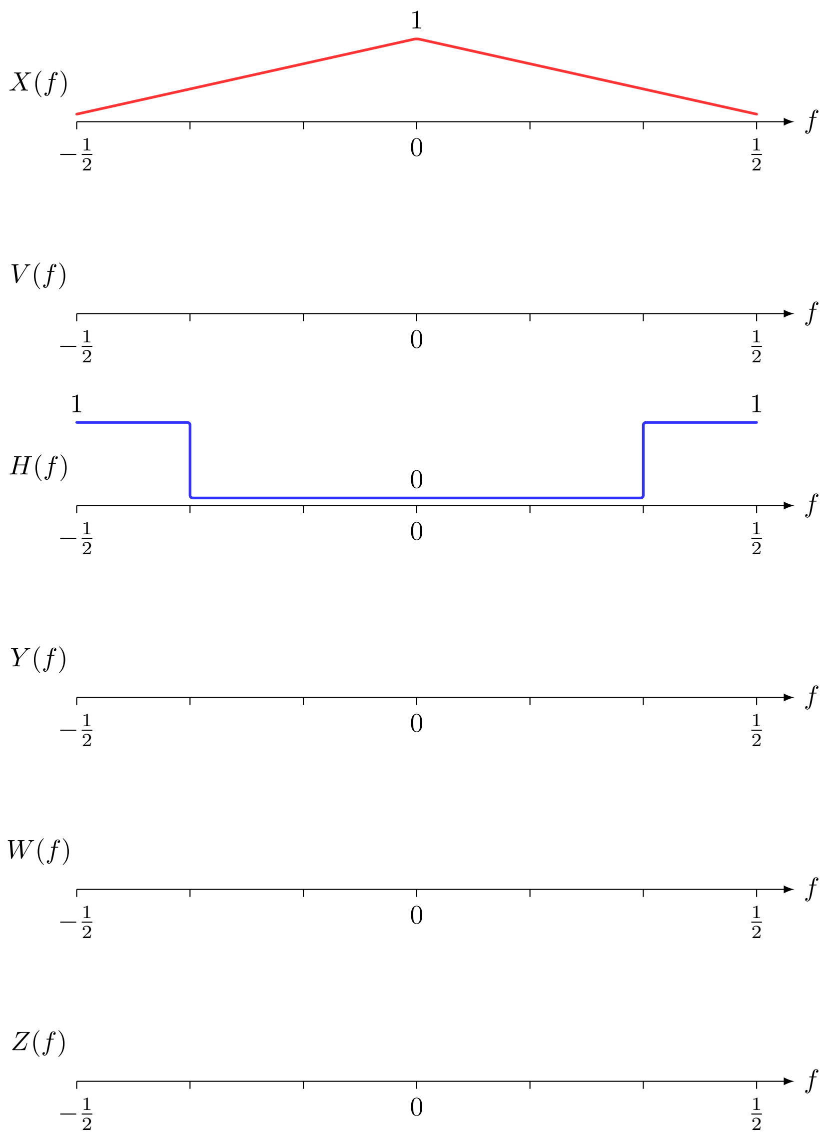


3. The signal $x[n]$ with spectrum (DTFT) $X(f)$ which consists of straight line segments as shown in the picture below is expanded by a factor $U = 3$ as shown in the diagram and then passed through a high pass filter with frequency response $H(f)$. The filter output $y[n]$ is decimated by a factor $D = 2$. On the axes provided, sketch $W(f), V(f), Y(f), W(f)$, and $Z(f)$. Note that $W(f)$ is the intermediate aliased signal internal to the decimator. Recall that the formulas for decimation are as follows:

$$W(f) = \frac{1}{D} \sum_{k=0}^{D-1} Y\left(f - \frac{k}{D}\right) \qquad \text{(aliasing formula)}$$

$$Z(f) = W\left(\frac{f}{D}\right) = \frac{1}{D} \sum_{k=0}^{D-1} Y\left(\frac{f-k}{D}\right) \qquad \text{(decimation formula)}$$





4. Consider the LTI system with frequency response $H(f) = j2\pi f$ for $-\frac{1}{2} \leq f < \frac{1}{2}$. Answer the following questions.

(a) Let $h[n]$ be the impulse response $h[n]$ of the system. Is $h[n]$ real, imaginary, or complex? Is it even, odd, or neither?

Real, imaginary, complex (circle one)
Even, odd, neither (circle one)

(b) Suppose the input to the system is an everlasting sinusoid $x[n] = \cos(2\pi f_0 n)$ where $0 < f_0 < \frac{1}{2}$. What is the output $y[n] = h[n] * x[n]$? Write your answer in the form $y[n] = A\sin(2\pi f_0 n)$.

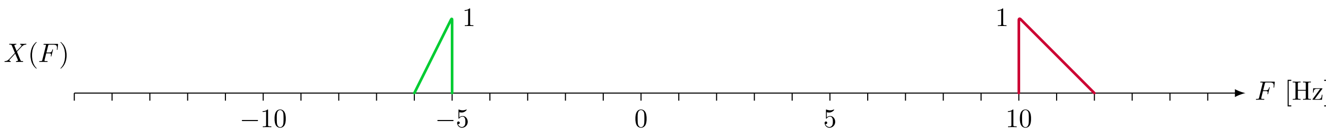
$y[n] =$

(c) Explain why this system has a differentiation effect $y[n] = \frac{d}{dn}x[n]$, even though derivatives are not defined for discrete-time sequences.

(d) Extra credit. Derive an expression for the impulse response $h[n]$ of the system with frequency response $H(f)$. Attach extra pages showing your work.

$h[n] =$

5. Consider the continuous time signal $x(t)$ with spectrum $X(F)$ shown below. Answer these questions.

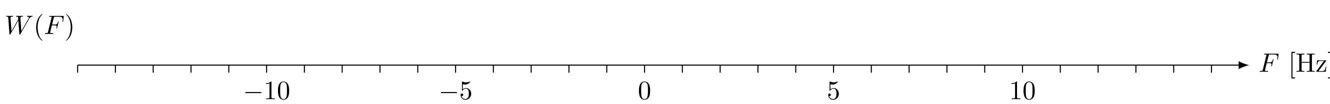


(a) The highest frequency in this signal is $F_{\text{max}} = 12$ Hz. The sampling theorem states that $x(t)$ can be reconstructed from samples taken at a rate $F_s > 2F_{\text{max}} = 2 \cdot 12 = 24$ samples/second. For the signal $x(t)$ given in this problem, we can sample at a rate below $2F_{\text{max}}$. What is the minimum sample rate F_s that avoids aliasing?

Minimum F_s =

(b) On the axis provided below, sketch the aliased spectrum $W(F)$. The aliasing formula is

$$W(F) = F_s \sum_{k=-\infty}^{\infty} X(F - kF_s).$$



(c) On the axis below, sketch the spectrum $Y(f)$ of the sampled signal. The formula for the sampled spectrum is

$$Y(f) = W(f \cdot F_s) = F_s \sum_{k=-\infty}^{\infty} X([f - k]F_s)$$